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Units and Dimensions in Physics

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The equations of physics are often interpreted as purely numerical relations. Endowing the symbols with "dimensional" quality as well as magnitude leads to a simpler, yet rigorous, mathematical structure, and permits changing units by direct substitution. Analysis of the mathematical universe of discourse of physical equations shows also that the four-dimensional *l, m, t, q* system of units is actually simpler than a three-dimensional system.

MUCH of the confused thinking about units and dimensions arises from an intuitive belief that dimensionality is a "pure" property of fundamental quantities of interest to the physicist, whereas units belong in the realm of numerical problems, or engineering. It is profitable to return to fundamentals, and see how the concepts of units and dimensions are associated with the mathematical representation of physical theories.

It is possible, though not practical, to describe physical laws by means of a minutely detailed description of the experimental procedure and measurement techniques. Various branches of physics went through this phase before appropriate theories were formulated in mathematical representation. The mathematical formulation of a law or theory was found to have many advantages, the most important of which are the precision of statement and the rigor of the rules of operation. Many physicists have taken advantage of only a portion of these properties; they consider a physical equation to be a semi-mathematical statement of a relation between quantities.[1–3] By this I mean that they conceive of an equation as being only numerically mathematical, and requiring context for precision. An equation such as $F=ma$ is interpreted as a relation between three numbers, valid when the numbers are obtained in a certain manner, in accordance with the units of measurement noted in the margin. To "improve" the situation, a constant of proportionality is added, so that $F=kma$, and a table of values of the pure number k appended for several sets of units.[4] The potential utility of mathematics does not become fully available so long as an equation is incomplete and must be qualified by marginal notes. Although recent textbooks[5] tend to consider the equations dimensional, many physicists[6] feel that this is a pedagogic device without fundamental justification.

[1] V. F. Lenzen, Am. J. Phys. **8**, 335 (1940).
[2] P. W. Bridgman, *Dimensional Analysis* (Yale University Press, New Haven, Connecticut, 1931), pp. 29, 41.
[3] A. G. Worthing, Am. J. Phys. **14**, 359 (1946).
[4] C. J. Coe, *Theoretical Mechanics* (Macmillan Company, New York, 1938), p. 16.
[5] E.g., E. Hausmann and E. P. Slack, *Physics* (D. Van Nostrand and Company, Inc., New York, 1948), third edition.
[6] See reference 2, p. 29 and references.

The obvious advance is from the above semi-mathematical physics to mathematical physics, wherein each symbol is completely defined, and does not change meaning in accordance with marginal notes. Thus the symbol m does not represent merely the numerical value assigned to a mass relative to some standard mass; the symbol m represents both the number and the concept of mass, that is, the operational algorithm by which a number is assigned to this particular mass. Thus, in the equation, we should not consider that $m=3$, when the mass is measured relative to the standard kilogram, but that $m=3$ kilogram; i.e., the symbol m comprises both a numerical and a nonnumerical part, each of which is subject to mathematical manipulation. The symbols thus represent elements in some appropriate mathematical universe of discourse.

Examination of the symbols or quantities of physics shows that there are several independent kinds. A distance, for example, has the attribute of length. Any unit in which a given distance can be expressed, centimeter, mile, angstrom, light-year, can be expressed as a numerical multiple of any other such unit. Thus we say that all units of distance have the same dimensionality, and that units of time have a different dimensionality. We leave it to the philosophers to interpret the dimensional concepts of length, time, etc. In physics, dimensionality is a relative matter, and we can state that two quantities (or units) are or are not of the same dimensionality without understanding the metaphysics of the dimensions. We indirectly define dimensions by specifying the experimental procedure to be used in measuring a quantity of a given dimension. The means of comparing quantities of the same dimension is the only "explanation" of dimension that is needed.

We now turn to the mathematical operations to be performed. We find that quantities of different dimensionality may appear in products or quotients, but they are never added to each other. On the other hand, quantities of the same dimensionality are subjected not only to multiplication and division, but also to addition and subtraction. Lastly, we find that the ratio of two quantities of the same dimensionality may undergo transcendental as well as algebraic manipulation. By the definition of "same dimensionality," such a ratio is a pure numeric. Thus in our mathematical universe we must define multiplication and division for all elements, addition and subtraction within certain subclasses of elements, and all operations appropriate to numbers for at least one subclass. Our universe of elements must therefore enjoy the group properties of multiplication, division, and the existence of the unit element. The universe must also contain the field of real numbers. Thus the elements having dimensionality may need only group properties.

If we designate any field element (pure number) by f_i, and any element not in the field by g_i, then the following operations (and their inverses) are allowed:

$$\begin{aligned} f_i+f_j&=f_k,\\ f_if_j&=f_m,\\ g_ig_j&=g_k,\\ f_ig_j&=g_m. \end{aligned}$$

The group elements can all be produced by operating on a minimal set of generating elements; any such set can be taken as our set of fundamental units, since any quantities of the same dimensionality are generated by the operation f_ig_i, and other quantities by g_ig_j.

We can easily *define* addition of quantities of the same dimensionality, since they can be expressed as products of numbers and a common g-element of that dimensionality:

$$f_1g_i+f_2g_i\equiv(f_1+f_2)g_i,$$

which already exists.

It is well known that physical equations are dimensionally homogeneous; they can always be written in the form $F(f_i,\ f_j,\ f_k,\ \text{etc.})g_m=0$. Since zero does not occur as a g-element, this equation requires $F(f_i, f_j, \text{etc.})=0$, or reduces to a purely numerical equation. It is this property that makes it possible to set up a consistent theory that the original equations are purely numeric. In fact, it is impossible to prove that this viewpoint is incorrect; it is simply cumbersome and unfruitful, for the useful technique of checking an equation dimensionally cannot be applied directly.

One important property of the concept of symbols as representing product elements fg, is the resulting ease of changing units. Thus we

interpret *mile* to mean identically $5280 \times ft$; not a count of 5280 separate *ft* measurements, but the element which is the product of the number *5280* and the element *ft*. We do not need an "explanation" of multiplication of a number by a unit, or of one unit by another, since all the rules of manipulation are well defined, and *physics* never requires this "explanation." We have given all the explanation we need when we call the ratio of a length element to a time element a velocity, or the product of distance and force, energy. That is, the physical equations in which products of units occur, *are* the definitions and explanations of the products. Our mathematical structure was chosen to be consistent with the operations encountered, and it also contains the rules of combined or repeated operations. It is always the application of the mathematical structure that defines the meanings of the elementary mathematical operations. The theory of groups, for example, can be applied to such diverse subjects as the study of permutations and the structure of braids. In each case the application defines the elementary operations; the mathematical structure supplies the rules and theorems for repeated or combined operations.

It is mathematically rigorous, therefore, to convert $v = 60$ mi/hr by substituting *5280 ft* for *mi*, and *3600 sec* for *hr*, finding

$$v = 60 \text{ mi/hr} = 60(5280 \text{ ft}/3600 \text{ sec}) = 88 \text{ ft/sec}.$$

There is an amazing variety of circumlocutions in the literature, purporting to justify the above direct substitution procedure. The concept that physical equations are numeric requires involved reasoning about measure-ratios, logometric theorems, square brackets, etc., to deduce that the answer given above is correct.[1,2,7]

We next approach the question of fundamental units. Many authors now use l, m, t, q as their primary dimensions; that is, the meter, kilogram, second, and coulomb are chosen as the generators for the group structure of our mathematical universe. Others object that this system is unnecessarily complicated, that l, m, and t suffice. Examining various quantities occurring in physics, we find that they can all be expressed in terms of l, m, t alone, provided half-integral powers of l and m are allowed. Some writers object that fractional powers of "dimensions" are meaningless, but the very physical laws that yield the fractional dimensions of certain physical quantities automatically define, or give meaning to, those fractional dimensions.

If we are to retain the simple group structure, we must take quantities equivalent to $\sqrt{l}$, $\sqrt{m}$, t as our generators. All the integral and half-integral power combinations can be generated by allowed group operations. Such a choice is sufficient mathematically, but very embarrassing physically, since we have no known physical quantities corresponding to our fundamental elements, hence cannot define our units in terms of standards of measurement.

The dilemma is easily resolved by taking l, m, t as the dimensions of the generating elements and *adding the root operation* to the nonnumeric part of our structure. Comparison of the so-called three- and four-dimensional unit systems on the basis of simplicity now depends on the relative simplicity of l, m, t, $\sqrt{}$, and l, m, t, q. It would intuitively seem that a fourth assumption of the same general kind as the other three would be simpler than assuming a new operation; on the other hand, the $\sqrt{}$ operation is already present in the numerical part of our structure.

A direct way of measuring simplicity is to examine consequences. The set of assumptions that generates fewer elements is simpler than the set that generates more; if we have enough, the additional elements are useless. The repeated operations of multiplication and division on either three or four generators yields an infinite set of elements, but a *denumerable* infinity. The elements can be placed in a one-to-one correspondence with the integers. It is easy to show, on the other hand, that including the $\sqrt{}$ operation yields a nondenumerable set of elements; they can be placed in a one-to-one-correspondence *with all the points of a line*. Hence, in a crude way, we can say that this latter set is infinitely larger than the former. Thus it appears that on both mathematical and aesthetic grounds, the four-dimensional system is simpler. Four-dimensional systems based on l, m, t, ϵ, or l, m, t, μ, also re-

[7] A. O'Rahilly, *Electromagnetics* (Longmans-Green Company, New York, 1938), Ch. 14.

quire roots, so there is little to be said in favor of these.[8]

There is still an infinite freedom in choosing which four elements shall be "basic." This is strictly a pun on *basic*, since we really mean the set of four on which we prefer to base our dimensional analysis, or our standards of measurement. For either purpose, the sets are equivalent. A physicist may, perhaps, prefer to analyze his equations in terms of l, m, t, q, and an engineer in terms of force, length, current, time—the FLIT system. Standards of measurement can be set for either one, no matter which we use. The ampere, for example, is (a current of) one coulomb per second. Either the ampere or the coulomb may be taken as "basic" by having its determination defined by absolute measurements. The other unit is then considered as a "derived" unit, but it is no less basic. The choice of which of these sets of four shall be adopted as (international) standards is primarily a matter of available experimental techniques, and has no effect on any individual's choice of basic units in either theoretical or practical work. Any two members of the coulomb-ampere-second triplet completely and uniquely determine the third.

There are no *fundamental* units to be found by a quasi-philosophical examination of nature; any standard reference system is a matter of convenience. Since a four-dimensional system is actually simpler than the classical three-dimensional system, there appear to be no logical grounds for adhering to the old ways. On the other hand, the historical system is logically valid, and familiarity is sufficient justification for its use by those so inclined.

The most cogent practical reason for the use of four-dimensional systems is the increased power and convenience of dimensional analysis. In fact, several writers have proposed the use of as many independent dimensions as possible. In the final analysis, the problem is a mixture of aesthetics and pragmatism, as is true of all physical theories.

[8] A three-dimensional system without roots can be generated from l, t, q, by defining permittivity to be a pure number. This system would probably be considered objectionable, because it does not allow mechanics to be treated as an independent subject.

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. **19**, 545 (1951)].

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Meson Masses and Energetics of Meson Decay*

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A series of measurements of the π- and μ-meson masses and their decay energies is reviewed. Recent mass measurements made at the University of California Radiation Laboratory have employed an improved method in which the ratio of the π-meson mass to that of the proton and the ratio of the μ-meson mass to that of the π-meson were determined by measuring the ratios of ranges and ratios of momenta for particles of equal velocities. The absolute decay momentum of the μ-meson was also determined in the same series of experiments. Other experiments at the Radiation Laboratory in which quite different methods were used to obtain the π-meson mass and the decay energy of the μ-meson are also discussed. The measurements are consistent with a π-meson mass of 276 electron masses and a μ-meson mass of 210 electron masses. In addition, no contradiction is found for the assumptions that the $\pi-\mu$ decay is accompanied by a neutrino and the $\mu-e$ decay is accompanied by two neutrinos.

OUR understanding of a physical process is only complete and satisfying when it is based on precise measurements. We have therefore included as a part of our program of meson research at the University of California Radiation Laboratory an investigation of the mass-energy relationships which exist between the π-meson, created in the primary production process, and its decay products.

It is well established[1] that the π-meson decays into a μ-meson and a neutral particle, which is probably a neutrino. The μ-meson in turn decays into an electron and presumably two neutrinos. The measurement of the meson masses has, of course, a certain intrinsic importance, but in addition the values of the masses enter into the total energies of the decay schemes, so that the mass measurements serve to confirm or contradict the assumed decay reactions.

At the beginning of our latest program of measurements, the situation was somewhat as follows: A series of experiments at the Radiation Laboratory starting with the work of Gardner and Lattes[2] had been made in which meson momentum and range in nuclear track emulsion were the quantities measured, and from which mass estimates were obtained. Each experiment had taught something new about measurement of the range and momentum, the control of background, limitations of the range-energy relation, range straggling, and the technique of using the cyclotron as a precision instrument. On the basis of a careful determination of the range-energy relation[3] for protons in Ilford C2 emulsion, additional meson mass measurements had been made from which we reported[4] the mass of the π-meson to be $276\pm6\ m_0$, where m_0 is the electron mass. We also gave the μ-meson mass as $210\pm4\ m_0$ based partly on older measurements of the π- to μ-mass ratio. Burfening, Gardner, and Lattes[5] had also measured the range in emulsion of a number of μ-mesons arising from the decay of π-mesons at rest. However, the particular emulsions in which the mesons were found had not been calibrated for stopping power, so that with the uncertainty in the μ-mass, only an approximate value of about 4 Mev was known for the decay energy of the μ-meson.

The experimental arrangement used in the earlier mass measurements can be understood by making reference to Fig. 1, which is an illustration of one of the first pieces of apparatus used for meson mass determinations. This "camera" is placed inside the vacuum chamber

* Invited paper VI, Meeting of the Am. Phys. Soc., Washington, D. C.; Phys. Rev. **83**, 236 (1951).

[1] Gardner, Barkas, Smith, and Bradner, Science **111**, 191 (1950). This is an article which reviews the production of mesons and their observed properties. It is recommended to readers who are not familiar with recent research in this field.

[2] Eugene Gardner and C. M. G. Lattes, Science **107**, 270 (1948).

[3] Bradner, Smith, Barkas, and Bishop, Phys. Rev. **77**, 462 (1950).

[4] Smith, Barkas, Bishop, Bradner, and Gardner, Phys. Rev. **78**, 86 (1950).

[5] Burfening, Gardner, and Lattes, Phys. Rev. **75**, 382 (1949).

of the cyclotron so that the circular orbits of the high energy alpha-particles (protons were later used) intersect the target which is in the form of a rod or ribbon of carbon or beryllium. Since the apparatus is in a strong magnetic field, the mesons produced at the target traverse curved paths and enter the surface[6] of a nuclear emulsion plate in which the mesons are detected. The mass of a positive particle can be determined using this apparatus if one calculates the meson momentum from the radius of curvature of its path and observes the range of the particle in the emulsion. These measurements suffice to estimate the mass if one has a range-energy curve applicable to the emulsion for a particle of known mass such as the proton.

The mass values quoted above were hardly those with which to be satisfied. We particularly regretted that the probable errors which we found it necessary to place on the measurements were much larger than the attainable statistical uncertainty. Effects leading to possible systematic errors included the following, as well as a number of minor ones:

(a) Variation of the stopping power of the emulsion from plate to plate and with the ambient humidity.

(b) Uncertainties in the absolute value of the cyclotron magnetic field, and in the position of the apparatus in the nonuniform field of the cyclotron. This problem had been partially solved by building a nuclear induction magnetometer and mapping the field in the vicinity of the apparatus, but a doubt still remained that these measurements were applicable under the conditions of the experiments. We now know, however, by further experience with these effects that we were unduly pessimistic in estimating the errors.

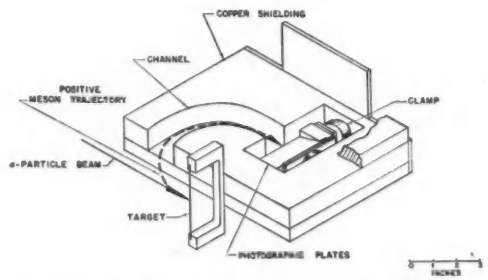

FIG. 1. An assembly used in one of the early meson mass measurements. The mesons leave the thin carbon strip target in the backward direction and enter through the surface of the emulsion which is mounted on a glass plate in the usual way. The plate is inclined a few degrees to insure that the mesons will enter through the surface. A channel is provided to limit the momentum interval accepted and to reduce background, but some mesons will scatter from the walls of the channel.

[6] Eugene Gardner, Phys. Rev. **75**, 1468 (1949).

THE NEW METHOD

A new approach has been taken during the past year in which a determined effort to eliminate the objections to the former measurements has been made. Our present method of mass measurement depends on the circumstance that if the range ratio of two particles of equal charge is found to be equal to the momentum ratio, their velocities are equal, and the common ratio is equal to their mass ratio. There are at least eight other measurable quantities which could be used in the same way for obtaining the mass of an unknown particle relative to that of a comparison particle, but range and momentum turn out to be the most convenient and accurate in this case.

By considering only momentum and range ratios we eliminate from the mass measurement the absolute value of the magnetic field intensity, and also any direct dependence on a range-momentum relation. The equations from which we deduce the meson masses are obtained by the following relativistically correct argument:

The range divided by the mass is a function only of the particle velocity. Similarly the momentum divided by the mass is a function only of the velocity. We can then write

$$R/M = f(P/M), \tag{1}$$

where R is the range, P the momentum, and M the mass, measured in units of the proton mass which is assumed to be well known. It turns out that the range-momentum relation[3] for protons of about 25 Mev is well described by a power law. In Fig. 2 it can be seen that the upper part of the curve is a surprisingly good straight line when plotted on log-log paper. Therefore for a small range of R and P we can write

$$R/M = C(P/M)^q, \tag{2}$$

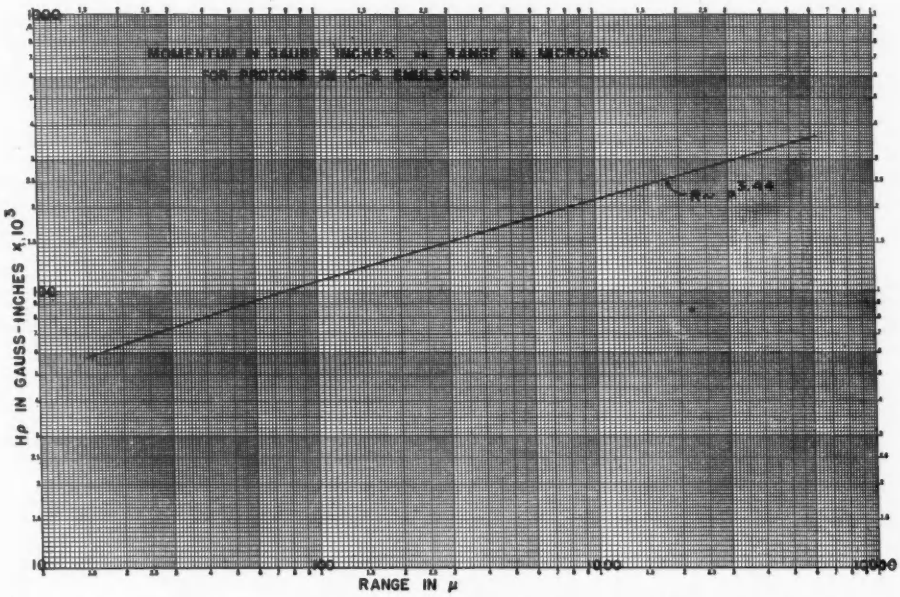

FIG. 2. The curve of log-momentum *vs* log-range for protons in Ilford C.2 emulsion. Note how straight the line is for ranges above 1000 microns. In this region the range-momentum relation is given accurately by the empirical law $R = Cp^{3.44}$.

where C and q are constants. However, the value of the constant C varies from plate to plate.

Our best estimate of q is 3.44. A very satisfactory consequence of our method of mass measurement is that little reliance need be placed on this number; the measurements are made in such a way as to eliminate almost entirely the dependence of the mass upon it. Constant C is evaluated by utilizing protons as comparison particles in the same emulsion in which the mesons are measured. Thus if we rewrite Eq. (2), solving for C and interpreting it for protons we have:

$$C = R_p/P_p{}^q, \text{ (proton mass is unity)}, \quad (3)$$

where R_p and P_p are the observable proton range and momentum, respectively.

We can then write Eq. (2) for the meson as follows:

$$M^{1-q} = (P/P_p)^{-q}(R/R_p)$$
(M in units of the proton mass), (4)

so that the meson mass can be calculated using Eq. (4) by measuring the momenta and ranges of mesons and protons in the same emulsion using the estimated value of q.

We now consider what happens when the meson and proton have the same velocity. In this case the meson-to-proton range ratio and momentum ratio are each equal to the mass ratio, M, and q disappears from the equation. Of course, we cannot obtain particles of identical velocities, but is is not difficult to match the velocities approximately and to select velocity intervals for protons and mesons which overlap almost completely. Then the derived mass is extremely insensitive to q. In fact, with reasonable errors in estimating q and in matching meson and proton velocities, the error in mass arising from these uncertainties cannot exceed a few parts in ten thousand.

To provide a source of protons of the correct momentum we introduce a second target into the cyclotron from which protons are scattered so that in the same emulsion we detect simultaneously protons and the various kinds of mesons—all of substantially the same velocity. To design the apparatus, we of course required a prior knowledge of the approximate mass ratios. Improvement of the meson yield relative to the background allowed the use of narrow momentum intervals.

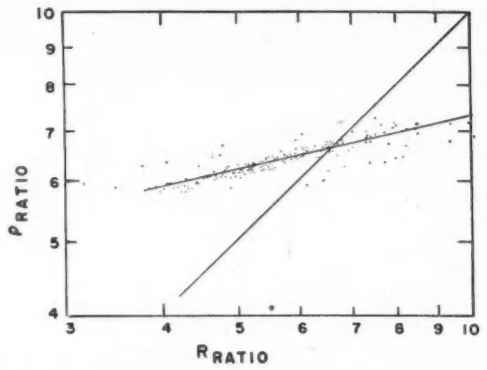

FIG. 3. A diagram showing the locus of points obtained when individual π-mesons are paired with protons. A point is plotted for each pair. The ordinate is the measured ratio of the proton to meson momentum, P_p/P, and the abscissa is the ratio of the proton to meson range, R_p/R. Where the locus crosses the 45° line the range ratio is equal to the momentum ratio, and this ratio is the mass ratio. This diagram was prepared from old data in order to obtain a large range of the variables; the later experiments utilize only data in the vicinity of the cross-over. Note the dispersion caused by range straggling. Some of the mesons doubtless come from points other than the target, so that a least squares straight line fitted to the locus probably does not yield the best mass ratio.

EXPERIMENTAL PLAN AND MICROSCOPIC MEASUREMENTS

To understand more fully the philosophy and arrangement of the meson-mass experiments, Figs. 3–5 may be helpful. In Fig. 3 the ratio of the proton momentum to the meson momentum was plotted against the ratio of the proton range to the meson range, using old data encompassing a large range of particle velocities. Since these data did not all come from the same plate, and some of the measurements had not been taken with the highest precision, one may use this diagram only to *estimate* the proton to meson mass ratio. Nevertheless the existence of a definite locus of points is clearly shown, and encouragement was provided to concentrate measurements in the region where the proton and meson velocities are equal. Figure 4 shows a plan view of the meson-to-proton mass ratio apparatus.[7] The apparatus was built to measure masses of both positive and negative π-mesons. μ^+ particles are also found in the plate. Some of these mesons originate in the target from the decay of π^+ mesons which stop in the target. Other μ-mesons are found in the plate which come from the decay of π-mesons in flight or at points other than the target. The apparatus was built to measure the mass of the μ^+ mesons coming from the target as well as the π-meson masses. Too much contamination by stray μ-mesons was found, however, so that a separate experiment[8] with somewhat modified apparatus was used to obtain the mass ratio, π^+/μ^+, and the momentum of the μ^+ meson coming from the decay of the π^+ meson.

Figure 5 is a more detailed diagram of the meson-to-proton mass ratio apparatus. The plate is mounted with the emulsion facing downward about an inch above the plane of the circulating proton beam in the cyclotron. Trouble was experienced with stray light, which tended to blacken the plate because no covering could be tolerated.

All mesons and protons enter the plate through the emulsion surface. Since the particle trajectory intersects the emulsion at a small angle to its surface, the entire range of the particle lies in the emulsion and can be measured accurately with an oil-immersion microscope. Tracks entering near the edge of the plate are not used, as there is a considerable amount of distortion at the edge arising from the shrinkage of the emulsion in processing. In determining the range,

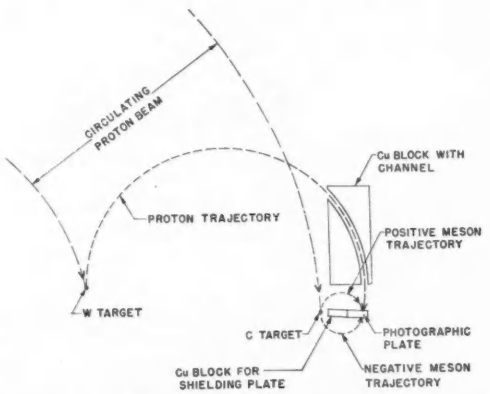

FIG. 4. Diagram illustrating geometry of experiment to measure meson to proton mass ratio. Scattered protons from the wolfram (tungsten) wire arrive at the plate with momentum greater than that of the mesons from the carbon target in about the ratio of their masses.

[7] Barkas, Smith, and Gardner, Phys. Rev. **82**, 102 (1951).

[8] Birnbaum, Smith, and Barkas, Phys. Rev. **83**, 895(A) (1951).

the lengths of the straight sections of the track between points where the meson suffers scattering by nuclei are individually measured and added. Allowance is made for the shrinkage of the emulsion perpendicular to its surface which occurs on the removal of the undeveloped silver halide. The emulsion shrinks to about 40 percent of its original thickness, depending to some extent on the ambient humidity both at the time of exposure and at the time of analysis.

STATISTICAL TREATMENT OF THE DATA

In calculating the meson-to-proton mass ratio we have adopted the following procedure: Individual determinations of that function of the mass in which the meson range appears linearly are calculated from the measurements using Eq. (4). Deviations from the mean are expected to be normally distributed and the arithmetic average is taken to be the most probable value. For this procedure to be correct, other statistical errors must be small in comparison with the range-straggling error, and the ranges of monoenergetic particles must be symmetrically distributed about the most probable range. Because of these assumptions, we found it important to investigate the phenomenon of range straggling rather closely. Figure 6 shows the distribution of μ^+ ranges found from π^+ mesons decaying at rest. Since it is believed that the π-meson decays into only two particles, the μ-meson and a neutral particle, conservation of energy and momentum requires that all the μ-mesons originate with exactly the same velocity. The distribution of ranges of the μ-mesons therefore provides an admirable example of range straggling. A chi-squared test[9] indicated that the distribution of the μ-meson ranges is consistent with an hypothesis of normality. The original theory of straggling given by Bohr leads to a standard deviation of 4.0 percent in ranges, assuming each electron in the emulsion to be free. We find a standard deviation only slightly larger than this. One particle range of only 267 microns was found by Miss F. M. Smith. This particle apparently does not belong to the main distribution, but it cannot be stated with certainty whether it arises from a different process of decay or whether the π-meson simply had not yet come to rest at the time it decayed.

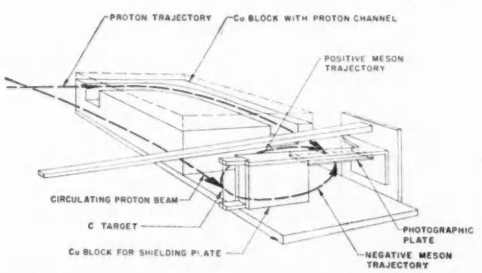

FIG. 5. Details of the meson target and plate holder. The target is in the form of a carbon cylinder 0.036 in. in diameter and a quarter inch high. It is mounted on a one-mil tungsten wire passing through it axially. Mesons spiral slightly upward from the target which is mounted in the median plane of the cyclotron, and enter the emulsion through the surface, which is facing downward. The larger copper block containing the proton channel shields the plate from stray protons.

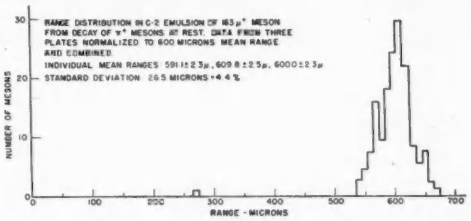
RANGE DISTRIBUTION IN C-2 EMULSION OF 183 μ+ MESON FROM DECAY OF π+ MESONS AT REST. DATA FROM THREE PLATES NORMALIZED TO 600 MICRONS MEAN RANGE AND COMBINED
INDIVIDUAL MEAN RANGES 591.1±2.3μ, 609.8±2.5μ, 600.0±2.3μ
STANDARD DEVIATION 26.5 MICRONS = 4.4 %
NUMBER OF MESONS
RANGE - MICRONS

FIG. 6. Histogram of ranges of μ^+ mesons. One meson with a range of 267 microns was found by F. M. Smith. This event clearly does not belong to the main distribution, and no evidence of inelastic scattering was observed in the track. An inelastic scattering event would, of course, reduce the meson range.

The results of our study of μ-meson ranges justifies the averaging procedure which we have employed.

ENERGETICS OF π-MESON DECAY

The mass-energy of a π-meson is converted into the rest mass and kinetic energy of its products when it decays. The π-meson decays into a μ meson and a neutral particle, and if it decays from rest the momenta of the two particles created are equal and oppositely directed. One of the questions which we wish to answer is whether or not the decay energy is consistent with a rest mass of zero for the neutral particle. The equation which must be satisfied to conserve energy and momentum is:

$$\nu^2/\mu^2 = 1 + (\pi^2/\mu^2) - 2\pi/\mu[1 + (p_0/\mu c)^2]^{\frac{1}{2}}, \quad (5)$$

[9] Paul G. Hoel, *Introduction to Mathematical Statistics* (John Wiley and Sons, Inc., New York, 1947).

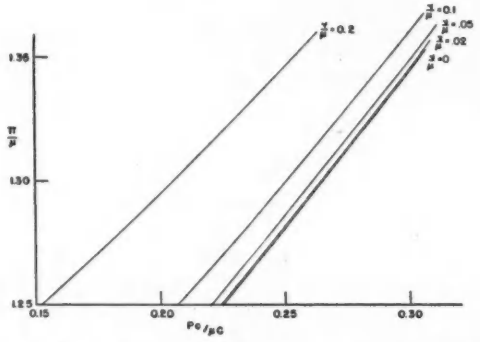

FIG. 7. The relation between the π- and μ-masses for various values of the neutral particle mass, ν, as a function of the momentum of the μ-meson. It can be seen that while our measurements are consistent with a scheme in which the π-meson decays into a μ-meson and a neutrino, the mass of the neutral particle can only be determined with an error of several times the electron mass.

where ν, μ, and π are the masses of the neutral particle, the μ-meson, and the π-meson, respectively, and p_0 is the momentum acquired by the μ-meson when the π-meson decays from rest. In Fig. 7 the connection between the ratios entering into Eq. (5) is shown. For the neutral particle to be a neutrino the measured ratios π/μ and $p_0/\mu c$ must lie on the curve $\nu/\mu=0$. It can be seen that extreme accuracy of measurement is required to distinguish between a neutrino and a decay particle having a rest mass equal to a few electron masses, but that little accuracy is required to distinguish a neutrino from a particle of rest mass equal to as much as one or two tenths of the μ-meson mass.

RESULTS

The first published observations[7] using the present method of mass measurement gave for the π-meson masses:

$$\pi^+=277.4\pm1.1\ m_0$$
$$\pi^-=276.1\pm1.3\ m_0.$$

We attach no significance to the difference in the results as the two values are separated by only about one unit of probable error. A difficulty was encountered in obtaining these results which has led us to try again with improvements in the geometry of the experiment. The mesons were found to be contaminated by particles which from their ranges could not have come from the target.

While we were able to measure the π-meson masses in spite of the stray meson background, it was found necessary to carry out another experiment[8] with similar apparatus in order to resolve the μ-meson peak and to obtain a good value for the π^+/μ^+ mass ratio. We reported a preliminary result of 1.317 ± 0.004 for this ratio.

In the same experiment we were also able to measure the decay momentum, p_0, of the μ^+ meson. This quantity was found as follows: First we find μ^+ mesons coming from the target having a momentum within a few percent of the full decay momentum. Measuring the momentum and range of these particles, we then extrapolate the momentum the small amount to match the range of decay mesons found in the emulsion. This requires a knowledge of the slope, q, of the log momentum-log range curve and of the absolute magnetic field intensity. The slope we know rather well from the combination of theory and measurement. The magnetic field was mapped by measuring the Larmor precession frequency of protons in the field using the so-called nuclear magnetometer. The calculation of the momenta of the mesons and protons observed is made from an analysis of their orbits in the magnetic field of the cyclotron. Figure 8 is an illustration of the type of path which is followed by a charged particle in the nonuni-

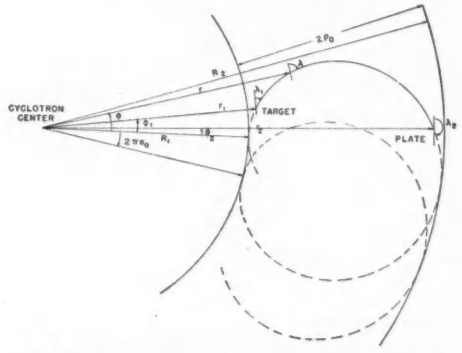

FIG. 8. Diagram illustrating the trochoidal type of orbit in the median plane followed by a charged particle in the radially decreasing magnetic field of the cyclotron. Careful calculations of the momenta were made from the measurements of the position and direction of the particle as it entered the emulsion. The trochoidal path arises from the fact that the radius of curvature is larger where the magnetic field intensity is lower. The orbit is periodically tangent to the libration limits at radii R_1 and R_2. All the quantities needed for an analysis of the orbit are labeled.

form field of the cyclotron. This method yielded a preliminary value of 29.86 ± 0.06 Mev/c for the meson momentum. The value of the μ^+ meson momentum together with the π^+/μ^+ mass ratio and the π^+ meson to proton mass ratio is consistent with the assumption that the neutral particle is a neutrino.

This experiment gives a number of other quantities of interest if we assume that the neutral particle is a neutrino. Thus we find the mass difference, $\pi^+-\mu^+$, in terms of the electron mass is $66.46\pm0.16\ m_0$, while the masses of the π^+ and μ^+ mesons are $\pi^+=276.1\pm2.3\ m_0$ and $\mu^+=209.6\pm2.4\ m_0$. In obtaining these figures no use has been made of the direct comparison of the π^+ meson and proton masses. The data also imply a value of 4.085 ± 0.044 Mev for the kinetic energy of the μ^+ meson.

OTHER EXPERIMENTS

In addition to our range and momentum method, additional means of measuring some of the quantities entering into the meson decay scheme have now been successfully employed at the Radiation Laboratory. The first of thse consists in balancing the equation:

$$p+p\rightarrow d+\pi^++Q$$

for the production of mesons by the collision of two protons. In this reaction the deuteron and π^+ meson are the only products. Therefore, using a monoenergetic proton beam the mesons observed in the forward direction will also be monoenergetic and will be the highest energy

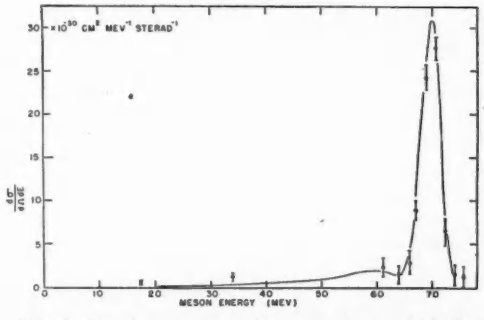

FIG. 9. The line spectrum of mesons from which Cartwright measured the π^+ mass. The ordinate gives the differential cross section for the reaction while the abscissa gives the energy of the mesons going in the forward direction. The energy of the proton beam was 340 Mev.

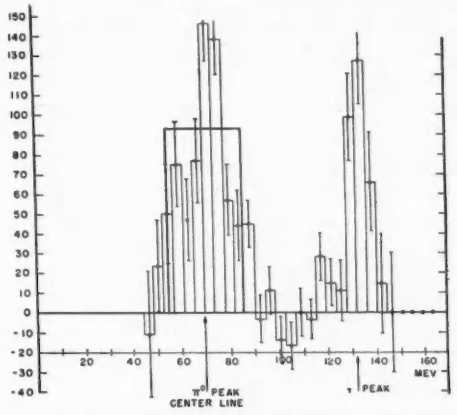

FIG. 10. Gamma-ray spectrum observed by Panofsky, Aamodt, and Hadley from the absorption of π^- mesons in hydrogen. The spectrum is constructed by observing the positron-electron pairs produced by the photons in a thin strip of tantalum. The high energy peak is attributed to the reaction $\pi^-+p\rightarrow n+\gamma$, and the low energy peak to the reaction $\pi^-+p\rightarrow n+\pi^0\rightarrow n+2\gamma$.

mesons obtained from the collision of two protons. A measurement of their range enables one to measure the meson mass and the value of the reaction energy Q. The line spectrum of mesons obtained from hydrogen was independently measured by V. Peterson *et al.*[10] and by Cartwright.[11] This method requires an accurate knowledge of the range-energy relation for charged particles in the material in which the mesons are stopped. The necessary data are available from the work of Bakker and Segrè[12] and the proton energy is known from the work of Mather.[13] Corrections must be made for the range straggling and for the geometrical straggling due to small-angle scattering. Their results are as follows: Peterson *et al.*, $279.0\pm1.5\ m_0$, and Cartwright, $275.1\pm2.5\ m_0$. Figure 9 shows the spectrum of mesons observed by Cartwright when the mesons were observed from the proton-proton reaction. The remarkable peak at the high energy end of the spectrum is its most striking feature.

Another beautiful experiment was carried out

[10] Peterson, Iloff, and Sherman, Phys. Rev. **81**, 647 (1951).
[11] W. F. Cartwright, Phys. Rev. **82**, 460 (1951).
[12] C. J. Bakker and E. Segrè, Phys. Rev. **81**, 460 (1951).
[13] R. L. Mather, Ph.D. dissertation, University of California, 1951.

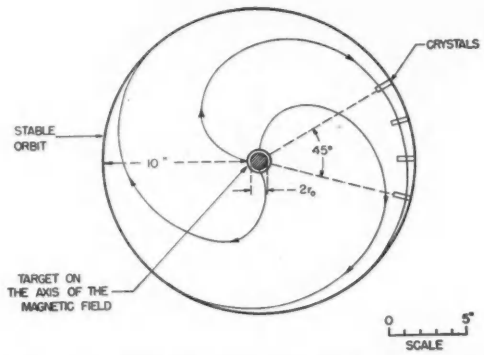

FIG. 11. Illustration of the spiral orbit spectrometer. The proton beam is directed perpendicular to the paper and traverses a target in the pole gap on the axis of the magnet. Electrons from the decay of μ^+ mesons stopping in the target follow paths as illustrated. Those which traverse all four crystals are counted. An important advantage of this type of spectrometer is the large solid angle of collection.

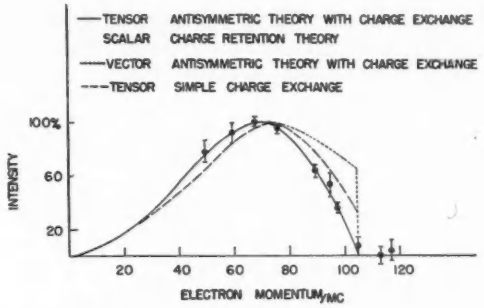

FIG. 12. The spectrum of positrons from the decay of μ^+ mesons. The points are the measurements, and the curves are the predictions of various theories. The upper limit of the positron energy distribution yields the μ-meson mass, if the meson is assumed to decay into a positron and two neutrinos.

by Panofsky, Aamodt, and Hadley[14] who measured the gamma-ray energy from the reaction discovered by them:

$$\pi^- + p \rightarrow n + \gamma.$$

This measurement yielded a π^- meson mass of $275.2 \pm 2.5\ m_0$. The experiment does not require a knowledge of a range-energy relation, but does employ an absolute magnetic field measurement. This was made by them with a nuclear magnetometer. Incidentally, the competing reaction,

$$\pi^- + p \rightarrow n + \pi^0$$
$$\pi^0 \rightarrow 2\gamma$$

gives a good estimate of the neutral π-meson mass. Panofsky *et al.* found:

$$\pi^- - \pi^0 = 10.6 \pm 2.0\ m_0.$$

The positron-electron pair spectrum from which the gamma-ray energies were obtained is shown in Fig. 10.

The general agreement obtained in these precision measurements of π-meson masses by entirely different methods not only gives us confidence in the correctness of the masses deduced, but also confirms our belief in the reliability of the range-energy relations used and the interpretation of the processes assumed in determining the masses. For example, that the charge on the meson is equal to the electronic

TABLE I. Measurements of meson masses and related quantities.[a]

Experiment	π^+	π^-	μ^+	T_μ	$\pi^- - \pi^0$	π^+/μ^+	p_0	$\pi^+ - \mu^+$
Smith, Barkas, Bradner, and Gardner	276±6	276±6	210±4					
Barkas, Smith, Gardner	277.4 ±1.1	276.1 ±1.3						
Cartwright	275.1 ±2.5							
Peterson, Iloff, Sherman	279.0 ±1.5							
Panofsky, Aamodt, Hadley		275.2 ±2.5			10.6 ±2.0			
Sagane, Gardner, Hubbard			212±5					
Birnbaum, Smith, Barkas Preliminary Data	(276.1 ±2.3)		(209.6 ±2.4)	(4.085 ±0.044 Mev)		1.317 ±0.004	29.86 ±0.06 Mev/c	(66.46 ±0.16)

[a] All masses in units of the electron mass. Quantities in parentheses have been calculated assuming neutral particle from decay of π-meson is a neutrino.

[14] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).

charge has been confirmed, because in the various experiments the charge enters in different ways and in each case the apparent mass is sensitive to the charge.

Finally we turn to a remarkable experiment carried out by Sagane, Gardner, and Hubbard.[15] For this experiment they utilized the large solid angle and good momentum resolution of the spiral orbit spectrometer to measure the spectrum of decay electrons from the μ^+ meson. Their experimental arrangement and results are shown in Figs. 11 and 12. The continuous distribution of electron energies found implies that the decay of the μ-meson is at least a three-body process. If the μ-meson decays into an electron and two neutrinos then the highest electron kinetic energy will be $(\mu-m_0)^2c^2/2\mu$. Using this assumption, the mass of the μ meson comes out to be 212 ± 5 m_0—an entirely independent determination of this quantity. The shape of the decay spectrum is of importance to meson theory, but does not directly affect the mass determination if the neutral particles are neutrinos.

All the results discussed in the present article are summarized in Table I.

This work was performed under the auspices of the Atomic Energy Commission.

[15] Sagane, Gardner, and Hubbard, Phys. Rev. **82**, 557 (1951).

Comic Dictionary of Physical Terms

Illustrated by MARGE M. MULLER
East Lansing, Michigan

Choke coil

Circle of confusion

Centripetal force

Cloud chamber

Shapes of Craters Formed in Plaster of Paris by Ultra-Speed Pellets*

JOHN S. RINEHART AND W. C. WHITE
Michelson Laboratory, U. S. Naval Ordnance Test Station, Inyokern, China Lake, California
(Received July 16, 1951)

Observations have been made on the craters produced in plaster of Paris targets by steel pellets that weighed 2 g and impacted at a velocity of 2.5 km/sec and by aluminum pellets that weighed $\frac{2}{3}$ g and impacted at a velocity of 4.7 km/sec. The angle of impact was varied from normal incidence to 75°. For angles of incidence between 0° and 45°, the mouth of the crater was substantially circular. At 60°, it had roughly the shape of an arrowhead. At 75° obliquity, the pellet ricocheted and produced an elliptical crater.

The area of the mouth of the crater in every case was many times the area of the impacting particle. The craters were characterized by a large bowl-shaped excavation from $\frac{1}{2}$ in. to 1 in. in depth with a shaft that extended a few inches into the plaster and whose area was approximately that of the pellet.

The volumes of the holes have been measured and related to the impacting energy and momenta of the respective pellets.

EXISTING meteorite craters have been described in detail by many authors.[1] Several of these authors have attempted to deduce from the shape, volume, and other topographical features of the crater the probable mass, velocity, and angle of impact of the meteorite that produced it. In a few cases, notably in studies of the lunar craters[1] and the Carolina Bays,[2] the shape of the crater has been used as a basis for affixing a meteoric origin to it. In general, the latter practice is dubious since there exist only a few craters that have been positively identified as being of meteoric origin. Even in the case of the established meteorite craters almost nothing is known about the velocities, masses, and angles of impact of the meteorites that made them.

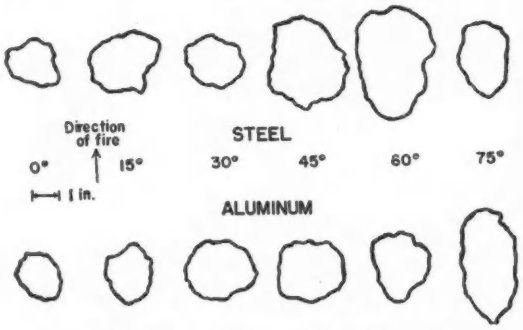

FIG. 1. Tracings of contours of the openings of craters excavated in plaster of Paris by aluminum and steel pellets.

In a previous paper[3] one of us summarized many of the data regarding the shapes of craters produced by high velocity missiles. The data in that paper were deficient in two respects. First, the velocities were relatively low, in the neighborhood of 1.6 km/sec, and second, all of the data referred to impact at normal incidence.

During the past several months we have been studying the nature of the craters produced in plaster of Paris targets by missiles impacting at velocities of 2.5 km/sec and 4.7 km/sec and at various angles of obliquity. While the velocities used in the present investigation are still considerably below the probable impacting velocities of many meteorites, it is believed that the results which we have obtained will be of considerable value in understanding the shapes of the craters that could be produced by meteorites.

EXPERIMENTAL PROCEDURE

Both steel and aluminum pellets have been fired. Both have an approximate diameter of one-half inch and are fairly irregularly shaped. The steel pellets were projected at a velocity of 2.5 km/sec and the aluminum pellets at a velocity of 4.7 km/sec. The mass of the steel pellet was 2.0 g. The mass of the aluminum pellet at the time of impact is not precisely known. Its orig-

* Work supported by Armament Branch, ONR.

[1] An excellent bibliography is given in R. B. Baldwin, *The Face of the Moon* (University of Chicago Press, Chicago, 1949).

[2] William Schriever, Trans. Am. Geophys. Union **32**, 87 (1951).

[3] J. S. Rinehart, Popular Astronomy **58**, 458 (1950).

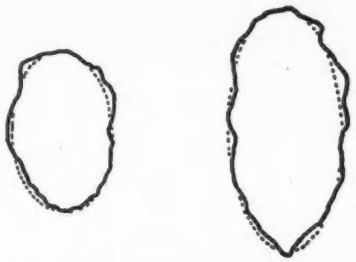

FIG. 2. Tracings of contours of the openings of trenches made in plaster of Paris by aluminum (left) and steel (right) impacting at an angle of 75°. Direction of fire was up. Dotted curves are true ellipses.

inal mass before projection was 0.7 g. However, during the impelling cycle it is believed to lose about one-tenth of a gram, so that in our calculations we have taken 0.6 g for the mass of the pellet.

The firing was done into large cast plaster of Paris blocks. Each block was approximately 15×15×7 in. The plaster was allowed to set some two weeks before the firings were conducted. At least three rounds were fired under each set of conditions. These blocks were large enough to insure that no edge effects came into play.

The angle of impact (defined as the angle between the normal to the surface and the trajectory of the pellet) was varied by 15 degree steps from 0° to 75°.

Each of the craters was examined in detail. The contour of the opening was traced and the area and diameter of the crater measured. The debris remaining in the crater was extracted and the volume of the crater determined by filling it with a measured volume of fine sand. Each block was sectioned along the plane containing the trajectory of the pellet.

CONTOUR OF CRATER MOUTH

Tracings of the contours of typical craters produced by steel pellets and by aluminum are reproduced in Fig. 1. At normal incidence the contour of the opening is usually fairly irregular although it is roughly circular. In looking at a large number of such contours one gains the impression that the openings are somewhat elongated in one direction. There appears, however, to be no logical explanation as to why this is so. Between 15° and 45° the contour of the opening remains nearly circular although some elongation becomes evident. The elongation is particularly evident in the case of the steel pellets. At 60° the shape of the opening is no longer circular, but has a distinctly arrowhead shape. The tip of the arrowhead points toward the direction from which the missile came. Beyond 60° the situation changes somewhat in that the pellet ricochets. The opening of the crater produced by a ricocheting pellet seems to be very nearly a true ellipse. The openings produced by both the aluminum pellet and the steel pellet at 75° angle of impact are reproduced in Fig. 2. An ellipse has been superimposed on each of the craters. The ratio of the major axis to the minor axis in each case is about 2 to 1.

It is interesting at this point to compare the shapes of the Odessa Crater and the Barringer Meteorite Crater with the present craters. The contours of the two craters have been drawn in Fig. 3. The contours on Monnig and Brown's topographical map[4] of the Odessa Crater have a distinct arrowhead appearance and remind one very much of the shapes of the craters produced at a 60° angle of impact. The Barringer Meteorite Crater is almost square in outline as has been pointed out by Zimmerman[5] If, however, one reorients Zimmerman's published photograph so that the southeast-northwest axis is vertical, he finds that the shape of the crater lies somewhere between the shape produced at a 45° angle of impact and a 60° angle. While the above comparison is somewhat convincing, we do not wish

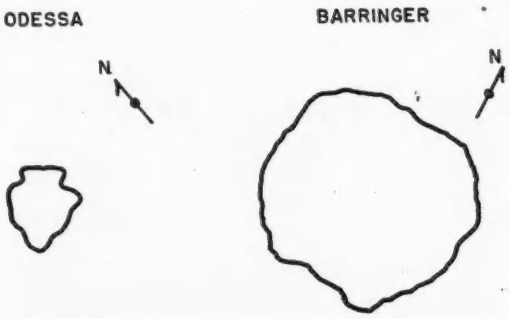

FIG. 3. Tracings of published contours of mouths of Odessa and Barringer Meteorite Craters. Two drawings are *not* to same scale.

[4] O. E. Monnig and Robert Brown, Contributions from Society for Research on Meteorites 1, 1 (1935).
[5] W. W. Zimmerman, Popular Astronomy 56, 496 (1948).

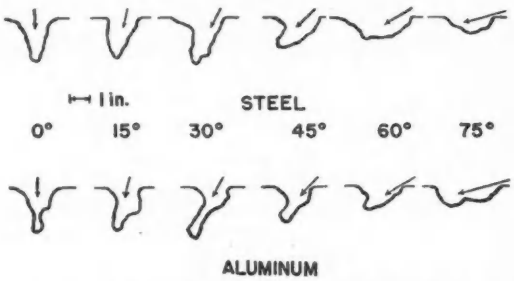

FIG. 4. Tracings of profiles of craters produced in plaster of Paris targets by impacts of aluminum and steel pellets.

to imply that we have unambiguously established the angle of impact for these two meteorites.

It is significant that the diameter of the opening is in every case much greater than the diameter of the impacting pellet. A tabulation of the data shows that both the area and the diameter of the mouth increase with increasing obliquity up to the point where ricochet begins. The increase is particularly noticeable with steel pellets but is not so pronounced in the case of the aluminum pellets.

PROFILE OF CRATERS

Tracings of typical crater profiles are reproduced in Fig. 4. Generally, the profile consists of a bowl-shaped excavation with a shaft that extends a considerable distance below the bowl. Although it is not immediately apparent from the figures, the shaft is almost an exact duplicate of the crater that is produced at normal impact. If, for example, one takes a 0° crater, tips it at an angle of 45°, and then lays it on top of the 45° crater, he will find that the two match almost exactly except for the lip of the bowl that extends to the left in the figures. The lip on the left arises from the fact that material is thrown out of the crater in a forward direction. One of the characteristic features of the profiles of the craters formed by the aluminum pellets is a small enlargement at the bottom of the shaft. We have been unable as yet to assign a definite cause to the origin of the enlargement. It could be due to vaporization of the pellet or to burning.

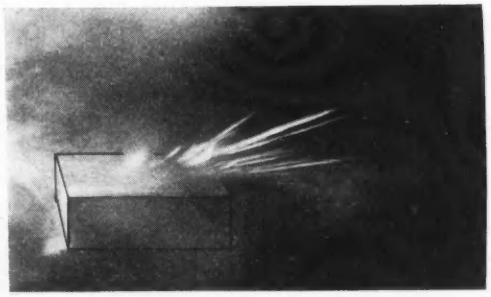

FIG. 5. Photograph of a ricocheting aluminum pellet. Firing from left to right. Angle of impact, 75°; velocity of pellet, 4.7 km/sec. Plaster block has been outlined to show detail.

For the 75° angle of impact there is no shaft. The missile strikes the target, digs a trench, and then flies off into space. Such a ricocheting aluminum pellet is shown in Fig. 5. It is obvious that the pellet shattered on impact and that the particles which fly off are burning vigorously.

As far as visible meteorite craters on the earth are concerned, one probably sees only the bowl-shaped excavation. For this reason we have in Fig. 6 filled the shaft with debris in order to see how the crater would appear if the shaft is obscured. When one does this he notes a rather surprising thing—namely, that one side of the crater is quite steep while the other side has a rather shallow slope. The steep side is on the right in the figures. This is the side from which the pellet came. In some of the discussions of the shapes of existing craters, investigators have assumed that the steep side of the crater corresponds to the side against which the missile impacted. To the extent that one can extrapolate present data to the impact of very large missiles, it would appear that we may have to reverse our thinking.

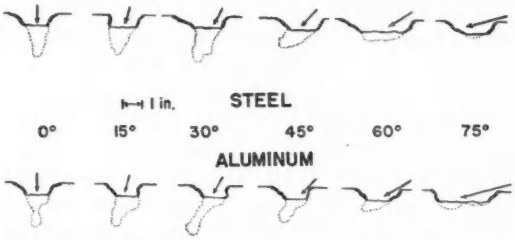

FIG. 6. Tracings produced in Fig. 4 modified by filling shaft with simulated debris.

In some cases the aluminum pellet broke up in flight before striking the target. Such behavior gave us an opportunity to study, to a limited extent, the kind of crater that would be produced

by the simultaneous impact of several particles. In Fig. 7 the profiles of a crater produced by an aluminum pellet that did not break up in flight and one that did break up in flight are compared. It will be noted that the diameter of the opening of the crater is roughly the same in both cases. In one case, however, a rather narrow shaft is drilled while in the other the crater is much more bulbous. The volumes of the two craters, however, were found to be substantially the same.

In a few cases we fired into plaster that had not yet completely hardened. An example of such a crater produced in such soft plaster is compared in Fig. 8 with one produced in hard plater. The principal differences between the craters and those produced in the hard plaster are (1) that their volumes are somewhat greater and (2) the depth of the bowl relative to the depth of the shaft is greater.

FIG. 7. Tracings of craters made by the impact of a single pellet and by the simultaneous impacts of several fragments of a pellet.

VOLUMES OF CRATERS

The measured volumes of the craters are plotted as a function of angle of impact in Fig. 9. At zero incidence the respective volumes of the aluminum and steel pellets are nearly the same. Their ratio is 1.06. The relative kinetic energy of the two, if we consider the aluminum pellet to have a mass of 0.6 g, is 1.06. Thus, at normal incidence it appears that the volume is proportional to the energy of the impacting pellet. It should be remembered here that while the two energies are nearly the same, the velocity of the aluminum pellet is nearly twice that of the steel pellet. The most interesting aspect of the data is that the volume of the crater increases with increasing obliquity up to a point where ricocheting occurs. Thus, the previously established relationship for proportionality between kinetic energy and crater volume which holds well for normal incidence breaks down at oblique incidence. It is believed that the increase in volume

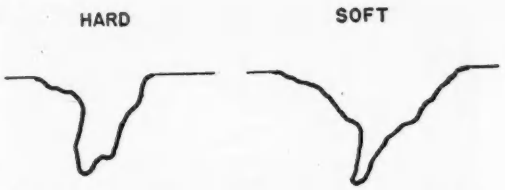

FIG. 8. Tracings of craters made in hard and soft plaster of Paris by steel pellets fired at an angle of impact of 30°.

is caused primarily by the ejection of material from the forward (left in Fig. 8) side of the crater. Ejection of this material probably represents momentum transfer rather than energy dissipation. It will be noted, for example, that the increase in volume with obliquity for the aluminum pellets is not so pronounced as it is for the steel. If momentum is the important parameter involved, then this is to be expected since the aluminum pellet has about two-thirds the momentum in the steel pellet.

DISTRIBUTION OF PELLET MATERIAL IN CRATER

During the present tests we have had an opportunity to observe how the pellet material is distributed within the crater. Usually in the case of steel the pellet was highly deformed but did not break up. This was particularly true at normal incidence and at 15° incidence. At 30° and 45° there was a greater tendency for the steel to shatter. When the steel shattered, the several pieces were distributed along the walls of the crater more or less at random. On the other hand, no pieces of the aluminum pellets were found in any of the craters. Occasionally a few pieces of aluminum would be found outside the crater on the surface of the plaster. We have assumed that these were pieces that probably bounced back out of the crater.

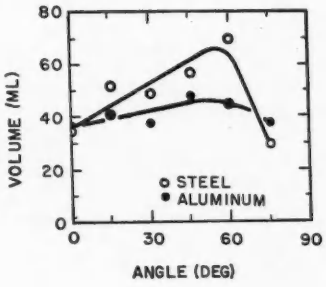

FIG. 9. Plot of measured volumes of craters against angle of impact.

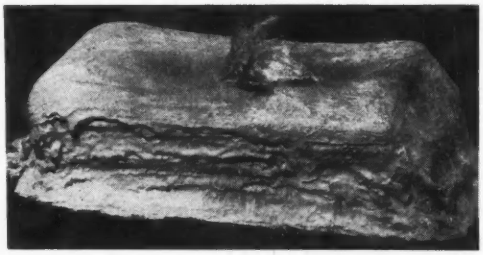

FIG. 10. External appearance of crater made in stratified plaster block by steel pellet. Angle of impact, 45°.

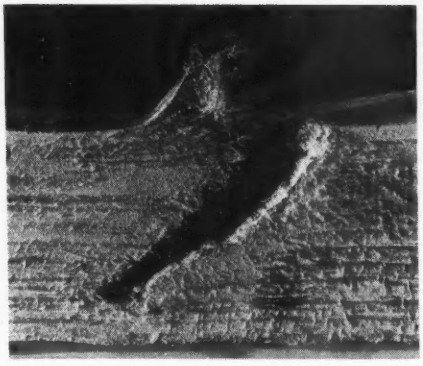

FIG. 11. Cross-sectional view of block shown in Fig. 10.

STRATIFIED MATERIAL

It has been recognized that homogeneous plaster does not completely simulate a stratified body such as the crust of the earth. Further testing in stratified materials is contemplated. In this connection we have fired one round into a block that was made up from surgical plaster dressing. A photograph of the external features of the crater produced in this block is reproduced in Fig. 10. The pellet was steel and the angle of impact was 45°. From this photograph one can see in a general sort of way the direction which the ejected material takes. A photograph of a sectioned view of the same block is reproduced in Fig. 11. The strata were dyed after the block was sectioned in order to bring out the displacement that took place. The strata in the region forward of the point of impact are considerably upturned while those behind the point of impact are relatively undisturbed.

CONCLUSION

In the foregoing discussion we have presented many data that relate to the shapes of the craters excavated in plaster of Paris by the impacts of ultra-speed aluminum and steel pellets. The velocities that we have been able to attain are still probably somewhat below the lower limit of the velocities of those meteorites that produced such craters as the Odessa and the Barringer. It is also recognized that the masses of the pellets we have been using are much less than those of the meteorites that excavated the craters. Because of these limitations great caution must be exercised in relating without qualification the present data to large meteorite craters. On the other hand, most phenomena in the field of terminal ballistics scale well, so extrapolation may not be completely unrealistic. At any rate, data of the kind presented here are beginning to reveal the fundamental nature of the mechanisms that enter into the formation of craters in rocklike materials by the impact of high speed missiles.

Oak Ridge School of Reactor Technology

About 70 students are to be selected by the Atomic Energy Commission's Committee of Admissions for enrollment in the Oak Ridge School of Reactor Technology for the session that begins September 8, 1952. Applications must be filed by March 1, 1952. Applicants must hold a Bachelor degree or a higher degree in chemistry, engineering, metallurgy, physics, or engineering-physics. Recent graduates are promised a stipend of $285.00 per month for the training period. Additional information about the school and application forms may be obtained by writing to the Oak Ridge School of Reactor Technology, P. O. Box P, Oak Ridge, Tennessee.

Physics Cooperates with Civilian Defense

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(Received July 19, 1951)

A report on the Radiological Monitoring Course offered as part of the Milwaukee Civilian Defense Program. In the first course 250 firemen, policemen, and people with technical background were trained in the principles of nuclear physics, and the effects, detection, and measurements of radiation. The course consisted of a series of lectures, together with appropriate films on the fundamentals of atomic and nuclear physics. The laboratory sessions consisted in familiarizing the trainees in the handling, reading, and care of three different radiation measuring instruments. Small radium sources were used. The field sessions consisted of the survey of a three curie cobalt-60 source at a local football stadium.

HISTORY records that an atom bomb was dropped on Hiroshima, August 5, 1945. In reality many bombs were dropped on August 5, 1945. One was dropped in the minds of all civilized men, and when the smoke and fire had cleared away, man suddenly realized how far and how fast science had gone in understanding the world in which he lived. A second bomb was detonated in the minds of all scientific men. The serene peace of the laboratory that Pasteur spoke about, and all research men enjoyed, was suddenly destroyed. Research was no longer limited to the universities, but governments all over the world were setting up facilities to continue the investigation and control of these new forces, not only for their military value, but especially for their peacetime applications. A third bomb was dropped in the minds of all federal, state, and municipal officials. These men of responsibility were suddenly presented with the problem of civilian protection, for the general know-how of atomic energy was no secret, and the possibility of atomic attack by a hostile nation would always present itself. The problems confronting municipal officials are serious and complex. They require the wholehearted support of the entire civilian population and especially the cooperation of the many physical sciences for their solution. In this paper, space permits only a short discussion of the part that physics can play in civilian defense, especially in the field of radiological defense.

TRAINING COURSE

The position of Radiological Defense in the Milwaukee Organizational Plan comes under the Division of Medical, Chemical, and Biological Defense as shown in Fig. 1.

To the best of our knowledge, this Radiological Monitoring Training Course[1] is one of the first of its kind in the nation conducted by any municipality for civil defense personnel. The course material was developed individually by Committee members[2] and reviewed by the Committee as a whole. The first five sessions of the Training Course consisted of the following lectures:

Lecture No. 1—Effects of Atomic Explosion.
Lecture No. 2—The Particles of Nuclear Physics.
Lecture No. 3—Basic Radiation and Detection Equipment.
Lecture No. 4—Biological Effects of the Atomic Bomb.
Lecture No. 5—Defense Against Atomic Bombs.

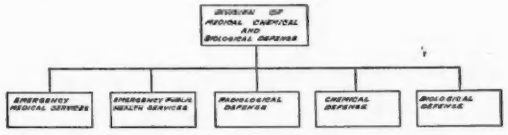

FIG. 1. Position of the Radiological Defense Committee in the Milwaukee civil defense plan.

[1] The City of New York started a Radiological Monitoring Training Course for policemen and firemen in the fall of 1950.

[2] The Radiological Defense Committee was made up of the following Physicists and Radiologists—F. R. Rehm and L. A. Penn, *Co-Chairmen, Milwaukee County;* Professors T. A. Rouse and W. J. Pearce, *University of Wisconsin at Milwaukee;* Professor M. Olson, *Milwaukee State Teachers College;* Professors A. G. Barkow and F. G. Karioris, *Marquette University;* Dr. H. W. Hefke, *Milwaukee Hospital;* and J. Wilson, *Allis-Chalmers Corpora-*

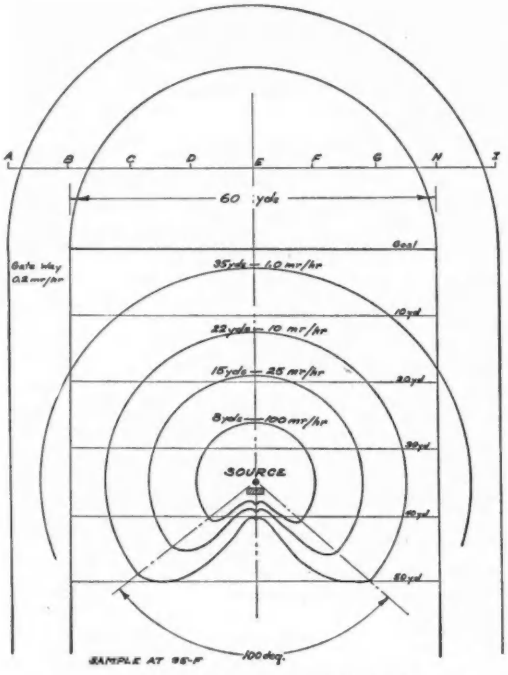

FIG. 2. The radiation plot of cobalt-60 showing four iso-dose lines.

Each lecture ran for a period of two and one-half hours with a fifteen-minute intermission. These lectures were augmented by various films on the subject. These included the following: Atomic Power, Atomic Physics, Atomic Energy, Medical Effects of Atomic Bomb, Radiological Safety, Atomic Medical Cases, Medical Services in Atomic Disaster, Pattern for Survival, Radiology Safety on Operation Sandstone.

The sixth and seventh session of the training course consisted of a laboratory session and a field exercise. The trainees were coached in the theory and operation of radiation instruments of the type using ionization chambers and Geiger-Müller tubes.

FIELD SESSIONS

The personnel taking part in the exercises were mostly firemen and will probably act as such in any disaster plan, their fire-fighting duties being of far more importance than their need as detailed radiological monitors. However, because it is felt that of all the civil defense personnel the firemen will be the first called upon to approach the bombed area, they should be the first line of approach in any radiological evaluation of the post-explosion condition. Following an atomic blast, the fire departments of the county will converge from their assigned deployment points or from their regular stations in an effort to control the tremendous fires in the devastated areas. With each department unit there must be a suitable radiation detection instrument and a competent person trained to interpret intelligently the radiation meter readings, lest these men be needlessly exposed. The radiation levels also determine the course of action—fire fighting, evacuation, rescue, or blasting fire lanes.

Firemen converging on the blast damaged area from all directions will be directed to report via existing portable communication means, the zones in which the mean sickness dose[3] will be exceeded in eight hours. They will clearly mark such areas and post them for the benefit of other civil defense personnel. Having marked and reported such areas to civil defense headquarters, they will proceed with their fire-fighting duties, entering the more radioactive zones only under the direction of the radiological officer of their company and for only the prescribed safe period of time.

From these reports civil defense headquarters can analyze the over-all radiological condition and advise the citizenry of affected areas. In addition more highly skilled professional monitors can then be dispatched to the radioactive areas to perform on-the-scene radiological evaluation of the hazards then existing. This group of personnel is now being enlisted and because of the highly technical nature of this subject will have to be high caliber professional men having a background in related fields. This group of people will perform such technical services as food and water monitoring, evaluation of hazards, decontamination, etc.

tion. These men also served as instructors for the lecture, laboratory, and field phases of the program.

[3] The "mean sickness dose" generally abbreviated MSD is the cumulative dosage which will cause half of the people exposed to it to become radiological patients. MSD has not been rigorously defined; it will vary with the rate of exposure. It may run as low as 100 r for very rapid exposures, and higher than 300 r for slow rates of exposures.

It may be of interest to point out the similarities between these exercises and the actual anticipated mode of operation in a disaster. It is expected that the fire departments will approach a bombed zone from all directions. It is probable they will leave their equipment to proceed on foot when nearing the ground zero point. When they reach a zone where the radiation meter indicates a hazardous condition exists, they will place radiation markers, dye, flares, etc., for the benefit of other civil defense personnel. In turn they will return to their trucks to use their radio equipment to warn civil defense headquarters of a dangerously radioactive zone. If their radio equipment is not functioning, they will then convey this information by means of any available communication method—amateur mobile radio, telephone, courier, etc. In order to take as little time as possible on the much-needed communication system, a concise method of reporting their information has been devised. Whereas today they report, "Jones-100 mr/hr-35 *F*-3:15 P.M.," they could just as easily be saying, "Jones-6 roentgen/hr N. 10th Street and W. Highland Avenue, 4-10-51; 3:15 P.M." The coordinate system of the football field simulates the coordinate system set up by our street intersections.

At the conclusion of the training course terms like roentgen, milliroentgens, curies, dose rate, iso-dose line, protons, neutrons, gamma-rays, etc., were as much a part of the fireman as taxes, baseball, and hamburgers are a part of the American way of life. In the field session the closest approach to an atomic bombing, i.e., from the standpoint of radiation, was simulated. A source of three curies of cobalt-60 was loaned by the A. O. Smith Corporation in Milwaukee for this experiment. The cobalt-60 was supplied by the Oak Ridge National Laboratory and is in the form of three pieces of cobalt wire $\frac{1}{8}$-inch long and $\frac{1}{8}$-inch in diameter arranged in the form of a triangle. To date it is the strongest source of gamma-radiation produced by the pile for industrial work. The proceedings took place on the football field in one of the city high school stadiums on the afternoons of April 3 and 10. The cobalt-60 was placed on the 35-yard line in the center of the field. To simulate irregularity of iso-dose lines in a devastated area, a concrete brick wall was erected about eight inches to one side of the radioactive sample.

A grid chart similar to the one shown in Fig. 2, but minus the iso-dose lines, was made available to each man. The group was divided into twelve teams of ten men each. Thirty instruments were available which made it possible to handle thirty men at a time. These consisted of ten Juno meters, ten Kelley-Koett meters of the ionization chamber type, and ten instruments of the Geiger-Müller type. The Juno meters and G-M instruments were loaned to the City of Milwaukee for this field session by the Chicago office of the Atomic Energy Commission and the Argonne National Laboratory. The duty of each team was to chart an iso-dose rate line of a given intensity about the source. These lines were identified by flags of different colors. The 100 milliroentgen per hour (mr/hr) line was indicated by red flags, 50 mr/hr by yellow flags, and 10 mr/hr by green flags. Each man on a team was required to chart five points on the iso-dose line, each time marking the point with a flag of the appropriate color, listing the coordinates of the point and the time of observation in his log book. After locating five such points the trainee returned to the information table set up on the track near the 50-yard line and used a walkie-talkie radio to report his findings to civil defense headquarters (Fig. 3). Civil defense headquarters for these exercises was set up in the press coop of the stadium. At headquarters a master radiological plot was

FIG. 3. A trainee reporting to field headquarters.

FIG. 4. The master plot at field headquarters.

made (Fig. 4). The report by the trainee to headquarters would run something like "Jones reporting 100 mr/hr at 35 F 3:15 P.M." and "Jones reporting 100 mr/hr at 40 D 3:20 P.M."

PROTECTION

Radiations from radioactive materials are dangerous and as such must be duly respected. The cobalt-60 sample was stored in a lead sphere ten inches in diameter and contained in an aluminum capsule as shown in Fig. 5. It was handled by tongs four feet long and always at arm's length (Fig. 6). The area was carefully surveyed by the instructors before the trainees were permitted on the field. In addition all

FIG. 5. The lead container for the cobalt-60.

FIG. 6. The cobalt-60 in position with reference to the brick wall.

instructors carried ionization chamber pocket dosimeters.

All trainees and observers who entered the stadium were issued film badges. This film in the presence of ionizing radiation will become exposed proportionally to the total amount of radiation received. From this it is possible to determine the radiation dose received by any individual having such a badge. There are obvious shortcomings in using this type of dose meter, mainly the delay between the radiation exposure and the film development plus the gigantic administrative task to keep badges and people straight. The delay between exposure and developing of the film may be disastrous.

To demonstrate the effect of radiation as a function of film blackening, test films were placed at various distances from the source. These were left in position until a total radiation dose of from 100 mr-10 r was obtained. The intensity of the radiation field was determined

TABLE I. Film blackening *vs* gamma-exposure.

100–1000 milliroentgens (Eastman code DF–7)		300 mr–10 roentgens (DuPont–552)	
Radiation	Blackening	Radiation	Blackening
Control	0.34	Control	0.24
100 mr	0.36	300 mr	0.33
200 mr	0.38	1 r	0.84
300 mr	0.40	3 r	1.29
400 mr	0.42	6 r	1.44
500 mr	0.44	7 r	1.48
600 mr	0.45	8 r	1.57
800 mr	0.52	9 r	1.59
1000 mr	0.57	10 r	1.64
Control	0.34	Control	0.25

by an ionization chamber survey instrument. All films were exposed for a period of three hours during the course of the exercises. For example, 300-mr film, which represents the weekly peacetime permissible dose[4] for a forty-eight hour week, was placed in the radiation field of intensity 100 mr/hr for three hours. The film blackening of these films is shown in Table I. All densities were measured on a Weston photographic analyzer. After developing approximately 400 film badges, not a single film was found that even approached the blackening caused by 100 mr of radiation.

The problems of civilian defense in the event of an atomic attack are many and complex. Citizens in all walks of life are being called upon to contribute time and effort toward this end. Similar to other work in the field of atomic weapons, the defense problem is being carried forward with the objectives of peace and security.

[4] Definitions of other radiation dosage terminology may be in order at this time. Whole body radiation of 400 roentgens is believed to be the dose which will kill 50 percent of the people exposed to it, and is known as the "mean lethal dose." Half of this amount, or about 200 r, would cause radiation sickness and might result in death for some individuals. Radiation of 600 r or more is lethal and would result in 100 percent deaths. For emergency operations, a person could be exposed to an "emergency dose" of 50 r of total body radiation without incurring serious injury and be able to continue at his duties. These figures are approximate and are used here only to show the probability of sickness and death. Some subjects may be quite sensitive to radiation and others quite resistant, so it is difficult to calculate the precise effects to be expected.

ACKNOWLEDGMENT

The authors wish to acknowledge with thanks the kind cooperation of the Milwaukee Sheriff Department in furnishing the pictures for Figs. 3–6.

A Laboratory Experiment with Coupled Linear Oscillators

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(Received May 7, 1951)

Two rolling masses and three helical springs are arranged collinearly in a horizontal plane. Longitudinal vibrations of the system are excited manually and the two normal modes studied. Calculated and observed frequencies can be compared in detail, since the coupling constant is directly measured. Agreement is obtained to within a few percent.

THE advanced undergraduate laboratory experiment described here has several purposes. First, the student is introduced to the idea of coupled oscillators. Second, he verifies quantitatively the rather unexpected consequences of a pair of simultaneous differential equations, solution of which is well within his grasp. Third, he is led to the application of some elementary theory of measurement in a meaningful situation in which both "calculated" and "observed" values carry probable errors.

Two small four-wheeled metal carts and three springs are arranged collinearly as shown in Fig. 1. In practice, the springs are about 5-inch lengths of window-shade-roller springs, preferably of somewhat different force constants k_1, k_2, and a, respectively. Two heavy screws are placed far enough apart on a 1×6 in. board so that each spring is stretched about twice its natural length. The carts are loaded unequally so that their total masses are m_1 and m_2, respectively. No special precautions are taken to reduce rolling friction.

Coupled oscillations of many different sorts may be obtained by moving one mass back and forth with various periods and amplitudes. Only the two normal modes are studied experimentally; it is shown in textbooks[1,2] that all other modes of vibration are linear combinations of these two.[3] It seems worth while to reproduce some of the theory here.

[1] Slater and Frank, *Introduction to Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1933), pp. 107–119.
[2] Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1948), second edition, pp. 52–66.
[3] This "superposition theorem" dates back to Daniel Bernoulli, *Hist. de l'Acad. de Berlin* (1753), p. 147.

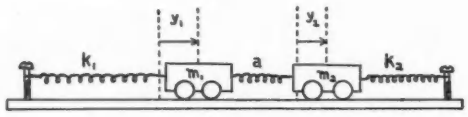

FIG. 1. A system of coupled linear oscillators.

Let the instantaneous displacements of the masses from their equilibrium positions be y_1 and y_2. Neglecting the masses of the springs, Newton's second law applied to each mass in turn gives

$$m_1(d^2y_1/dt^2) = -k_1y_1 - a(y_1 - y_2), \quad (1a)$$

$$m_2(d^2y_2/dt^2) = -a(y_2 - y_1) - k_2y_2. \quad (1b)$$

For convenience,[4] set $y_1m_1^{\frac{1}{2}} = x_1$, $y_2m_2^{\frac{1}{2}} = x_2$, $(k_1+a)/m_1 = \omega_1^2$, $(k_2+a)/m_2 = \omega_2^2$, $a/(m_1^{\frac{1}{2}}m_2^{\frac{1}{2}}) = c$. (Note that ω_1 is the angular frequency, $2\pi\nu_1$, which m_1 would have if m_2 were held fixed, and similarly for ω_2. The constant c measures the amount of coupling between the masses.) Equations (1) then become

$$d^2x_1/dt^2 + \omega_1^2x_1 - cx_2 = 0, \quad (2a)$$

$$d^2x_2/dt^2 + \omega_2^2x_2 - cx_1 = 0. \quad (2b)$$

The normal modes, in which we are interested, are those in which the two masses vibrate sinusoidally, with the same frequency. Moreover, we look for solutions in which m_1 and m_2 vibrate either exactly in phase or exactly out of phase. Therefore, we assume

$$x_1 = A \sin\omega t, \quad (3a)$$

$$x_2 = B \sin\omega t, \quad (3b)$$

where A and B are both real. For the "in-phase" mode, A and B have the same sign, while for the "out-of-phase" mode they have opposite signs. Substitution of Eq. (3) into Eq. (2) yields

$$(\omega^2 - \omega_1^2)A + cB = 0, \quad (4a)$$

and

$$cA + (\omega^2 - \omega_2^2)B = 0. \quad (4b)$$

Rearranging and dividing gives a quadratic equation in ω^2:

$$\omega^4 - (\omega_1^2 + \omega_2^2)\omega^2 + \omega_1^2\omega_2^2 - c^2 = 0. \quad (5)$$

Solving for ω^2, we get *two* angular frequencies, which are the normal modes:

$$\omega'^2 = \tfrac{1}{2}(\omega_1^2 + \omega_2^2) + \tfrac{1}{2}[(\omega_1^2 - \omega_2^2)^2 + 4c^2]^{\frac{1}{2}}, \quad (6)$$

$$\omega''^2 = \tfrac{1}{2}(\omega_1^2 + \omega_2^2) - \tfrac{1}{2}[(\omega_1^2 - \omega_2^2)^2 + 4c^2]^{\frac{1}{2}}. \quad (7)$$

The ratio A/B may also be found, and straightforward analysis shows that ω', the solution of higher frequency, represents the out-of-phase mode for which $A/B < 0$, while ω'' represents the in-phase mode for which $A/B > 0$. It follows from elementary theory of equations applied to Eq. (5), as well as from addition of Eqs. (6) and (7), that

$$\omega'^2 + \omega''^2 = \omega_1^2 + \omega_2^2. \quad (8)$$

The experiment consists of verifying Eqs. (6)–(8).

Because of frictional damping, it is hardly feasible to measure the frequencies except under conditions of sustained vibration, in which energy losses are compensated by externally supplied impulses timed to one of the natural frequencies of the system. A seemingly crude manual excitation of the vibrations is used, with surprisingly good results. One mass is grasped lightly and given a suitable small shove once during each vibration, the hand and therefore the mass being kept in approximate simple harmonic motion. The normal mode is recognized by the criterion that the other mass, acted on only by the springs, also executes approximate simple harmonic motion. When vibrating correctly, the second mass exerts no sudden reactions on the first one. With a little practice, a normal mode can be maintained indefinitely, muscular sensations being combined with visual ones in the process of maintaining resonance.

Each of the four angular frequencies ω_1, ω_2, ω', ω'' is determined from the mean of ten observations of the time for 20 vibrations. In the determination of ω_1, the mass m_2 is held fixed in its equilibrium position, and similarly for determination of ω_2. Equation (8) may be verified without further measurement. To verify Eqs. (6) and (7), the force constants of the springs and the masses must be found. The force constants may be determined by any standard method in which the spring is suspended vertically and weights added. An instructive method is to plot total loads against scale readings, and then find the slope of the least squares line, to-

[4] We use the notation of reference 1.

gether with the probable error[5] of the slope. If less elaborate averaging methods are used, the order of magnitude of the pe may be estimated for each force constant, from internal consistency. The masses are determined by weighing.

The following data indicate the sorts of results which may be expected. The indicated errors are internal[6] pe's for the observed ω_1^2, ω_2^2, ω'^2, ω''^2; estimated pe's for k_1, k_2, a; and limits of error for m_1 and m_2. The other pe's are found from the usual rules for propagation of errors.

m_1 952 ± 0.5 g
m_2 1164 ± 0.5 g
k_1 $2.80\pm0.03\times10^4$ dyne cm^{-1}
k_2 $2.14\pm0.03\times10^4$ dyne cm^{-1}
a $2.54\pm0.03\times10^4$ dyne cm^{-1}.
The coupling constant c is calculated to be 24.2 ± 0.3 rad^2 sec^{-2}.
The following are expressed in rad^2 sec^{-2}:

ω_1^2	calc.	56.0 ± 0.5
	obs.	55.0 ± 1.3
ω_2^2	calc.	40.2 ± 0.4
	obs.	42.5 ± 1.8
ω'^2	calc.	73.5 ± 0.4
	obs.	75.6 ± 4.4
ω''^2	calc.	22.7 ± 0.4
	obs.	25.5 ± 2.5
$\omega_1^2+\omega_2^2$	calc.	96.2 ± 0.6
	obs.	97.5 ± 2.2
$\omega'^2+\omega''^2$	calc.	96.2 ± 0.6
	obs.	101.1 ± 5.1

A good workout in the principles of combination of errors is provided in the experiment. All of the ordinary operations are involved, including sum, difference, product, quotient, power, root, and arithmetic mean.

For a shorter experiment, all calculation of frequencies may be omitted and Eq. (8) verified using observed frequencies only. For the capable student, the experiment can be extended along theoretical lines. For instance, when the out-of-phase mode is excited, a point on the connecting spring is observed to remain stationary. The student may be asked to show that this point divides the connecting spring into two segments, the ratio of whose lengths is $m_1(\omega'^2-\omega_1^2)/a$. He may also show that the effect of coupling is always to spread the frequencies apart so that ω' is greater than either ω_1 or ω_2 while ω'' is less than either ω_1 or ω_2.

Experiments with coupled pendulums have been described in many places.[7] The present experiment, while perhaps more elementary, has the advantage that the coupling force can be readily measured, being of the same nature as the forces giving rise to the uncoupled oscillations. In pendulum experiments, the coupling constant is usually computed from the measured periods, direct measurement being difficult or impossible. Searle[8] has described an experiment which is the exact equivalent in torsional motion of the present experiment. Because of the small damping, the free vibrations of the system may be studied. Searle's experiment is capable of yielding precision of the order of a fraction of a percent, but the theory is necessarily more complicated than that for linear vibrations. It is felt that the linear experiment strikes a happy compromise with respect to simplicity of theory and precision of results.

[5] Birge, Phys. Rev. **40**, 202–227 (1932).
[6] For an arithmetic mean of several equally reliable observations, the only sort of pe that is meaningful is the internal pe, based on internal consistency. See reference 5, p. 214. The pe's of the observed ω_1^2, etc., are calculated from the internal pe's of the observed times for 20 vibrations.
[7] E. g., Olson, Am. J. Phys. **13**, 321–324 (1945).
[8] Searle, *Experimental Harmonic Motion* (Cambridge University Press, London, England, 1922).

But nothing has been done conducing directly to the encouragement of science in general. . . . The protection and favour of the Crown is still bestowed only upon those pursuits which are immediately connected with the defence or government of the country, or with the promotion of wealth or luxury, without reference to intellectual cultivation or the advancement of knowledge. Science, however, ever has been, and ever will be, of too elastic a nature not to make progress under every discouragement.—F. R. S., "Thoughts on the Degradation of Science in England" (1847). Quoted by GEORGE A. FOOTE, *Isis* **42**, 206 (1951).

Quantitative Evaluation of Rocket Propellants

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A method for the quantitative evaluation of chemicals as rocket propellants is described. The procedure utilizes the fact that adiabatic expansion through a nozzle may be considered to be isentropic. Treatments are presented for the two limiting conditions of expansion, corresponding to flow without chemical change and to flow which is characterized, at all times, by the existence of thermodynamic equilibrium.

A DETAILED account of the physics of rockets has been given in a series of papers by Seifert, Mills, and Summerfield, who have, however, not discussed the quantitative evaluation of propellant performance.[1] In view of the fact that one of the major contributions made by physical chemists to the development of improved jet engines is the result of excellent correlation between observed performance and performance calculated by utilizing the basic laws of thermodynamics, it is of evident importance to consider these evaluation techniques in some detail. Accordingly, the present discussion will be devoted to a review of sound methods for the theoretical calculation of propellant performance, assuming only familiarity with the principles of thermodynamics.[2]

I.

Before describing methods for evaluating the performance of chemical propellants in jet engines, it is desirable to review the basic physical equations which relate the thrust on a rocket engine to the physico-chemical characteristics of the propellants. In particular, it will be shown that with some simplifying assumptions the performance of a given propellant system depends only on the absolute pressure in the combustion chamber and on the absolute external pressure into which the combustion products exhaust. A diagram depicting a cross section of a rocket engine is shown in Fig. 1.

From the equation for conservation of momentum it can be shown[3] that the thrust F on a rocket motor is

$$F=(\tfrac{1}{2}+\tfrac{1}{2}\cos\alpha)\dot{m}v_e/g+(p_e-p_0)A_e, \qquad (1)$$

where α = half of the divergence angle of the nozzle; $\dot{m}$ = total weight-rate of flow; g = acceleration of gravity; v_e = linear flow velocity at the nozzle exit; p_e = absolute pressure at the nozzle exit; p_0 = external atmospheric pressure; and A_e = cross-sectional area at the nozzle exit. If the *effective exhaust velocity* c is defined by the relation

$$c=(\tfrac{1}{2}+\tfrac{1}{2}\cos\alpha)v_e+(p_e-p_0)A_e/(\dot{m}/g), \qquad (2)$$

then Eq. (1) becomes[3]

$$F=\dot{m}c/g. \qquad (3)$$

Maximum thrust is obtained under conditions in which the exit pressure p_e equals the external pressure p_0,[3,4] a condition which can be met for any given value of p_0 by proper construction of the Laval nozzle. If $p_0=p_e$ and, furthermore, α is made sufficiently small, then

$$c=v_e, \quad F=\dot{m}c/g=\dot{m}v_e/g \qquad (\alpha \text{ very small}, \ p_0=p_e). \qquad (4)$$

A convenient measure of propellant performance is the thrust per unit weight-rate of flow, which is called the *specific impulse* and is designated by the symbol *Isp*. From Eq. (4) it

[1] Seifert, Mills, and Summerfield, Am. J. Phys. **15**, 1, 121, 255 (1947).

[2] No attempt will be made to give proper credit to the many scientists in this country and abroad who have used or developed independently methods which are substantially equivalent to those described in this article. Among the important early works on jet propulsion principles published in this country, in which the evaluation of propellant performance is discussed, may be mentioned two compilations, one edited by H. S. Tsien and published by the Guggenheim Aeronautics Laboratory of the California Institute of Technology in 1944, and the other edited by B. L. Crawford, Jr., and published under the auspices of George Washington University in 1944.

[3] H. S. Tsien and F. J. Malina, J. Aeronaut. Sci. **6**, 50 (1938).

[4] F. J. Malina, J. Franklin Inst. **230**, 433 (1940).

is apparent that

$$Isp = F/\dot{m} = c/g, \tag{5a}$$

and

$$Isp = v_e/g \ (\alpha \text{ very small}, p_0 = p_e). \tag{5b}$$

The dimensions of the specific impulse are evidently those of time, *viz.*, lb thrust times sec per lb weight, grams thrust times sec per g weight, etc. It should be noted that in the derivation of Eq. (5a) we have used only the equation for conservation of momentum with the assumption that v_e is parallel to the nozzle axis and has the same value at every point in the plane of A_e. Equation (5b) involves the additional assumptions that α is small and that $p_0 = p_e$.

In order to utilize Eqs. (5a) and (5b) for approximate practical evaluation of motor performance it is necessary to introduce simplifying assumptions. The simplifying assumptions which are usually made are the following:

Assumption (1): Thermodynamic equilibrium is reached in the combustion chamber after adiabatic reaction.

Assumption (2): Expansion of the combustion products through the Laval nozzle is adiabatic.

Assumption (3): The products of combustion behave as ideal gases.

Assumption (4): The expansion process may be considered to involve one-dimensional flow (parallel to the nozzle axis) of nonviscous ideal gases.

Assumption (5): The velocity of the gases at the nozzle entrance position is negligibly small compared to that at the exit position.

Experience has shown that each of the five listed simplifications introduces only small errors into the calculated values of *Isp*. Furthermore, for a great number of useful propellant combinations, the errors are of similar magnitude, whence it follows that Eq. (5a) or Eq. (5b) can be used together with assumptions (1) to (5) to obtain a reliable estimate of the *relative* performance of propellant systems.

It should be noted that theoretical estimates of *Isp* are not possible at all if thermodynamic equilibrium is not reached in the combustion chamber, unless the deviations from equilibrium

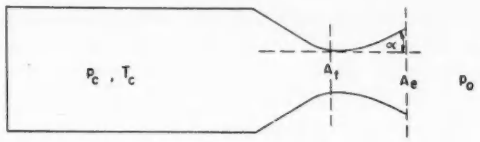

FIG. 1. Cross-sectional drawing of a rocket engine: p_c = pressure in combustion chamber; T_c = temperature in combustion chamber; A_t = cross-sectional area at the nozzle throat; A_e = cross-sectional area at the nozzle exit; and p_0 = external pressure.

are specified precisely. Our present knowledge of combustion processes is insufficient to permit *a priori* calculations of the approach to thermodynamic equilibrium in a given combustion chamber. The ultimate justification for assumption (1) is, therefore, the agreement between theory and experiment which is obtained for the results derived from assumption (1).

The approach to ideal gas behavior of the combustion products can be calculated by using a suitable equation of state for gas mixtures. Detailed calculations show that the assumption that the combustion products behave as ideal gases introduces small errors, of the order of 1 percent or less, into the calculated values of the *Isp* for conventional jet engines. For very high pressure applications, e.g., liquid-propellant guns, it is, however, very important to correct for gas imperfections.

The expansion of the combustion products in the Laval nozzle is never completely adiabatic, although heat transfer to the nozzle will introduce only small errors (of the order of a few percent or less) into the calculated values of *Isp* for rocket engines of reasonable size.

The remaining approximations, which are introduced through assumptions (4) and (5), also introduce negligibly small errors into the calculation of *Isp* for conventional rocket motors.

Using assumptions (1) to (5), it follows from the first law of thermodynamics that

$$\tfrac{1}{2}v_e^2 = \Delta h_c^e, \tag{6}$$

where Δh_c^e represents the enthalpy change per gram accompanying expansion of the combustion products from the chamber temperature T_c to the exit temperature T_e. The proper evaluation of Δh_c^e, even for ideal gases, requires careful considerations of the chemical composition changes which occur during expansion through

the nozzle. It is apparent by reference to Eqs. (5a) or (5b) and Eq. (6) that the evaluation of performance of chemical propellants reduces essentially to the calculation of enthalpy change during adiabatic expansion.

Although the average heat-capacity ratio of a gas mixture, $\bar{\gamma}$, is a function of temperature for all propellant systems, and, furthermore, the average molecular weight of the gas mixture $\bar{M}$ will change during the expansion of the combustion products for all systems except those for which no chemical reactions of any kind occur during expansion, both $\bar{\gamma}$ and $\bar{M}$ are usually not very sensitive functions of temperature. For this reason it has become customary to obtain approximate expressions for v_e and T_e which are based on the following two additional assumptions:

Assumptions (6) and (7): A suitable average heat capacity ratio $\langle\bar{\gamma}(T)\rangle_{Av}=\delta$ and a suitable average molecular weight $\langle\bar{M}(T)\rangle_{Av}=\bar{M}^*$ can be assigned for the combustion products which are expanding adiabatically from T_c to T_e.

It is evident that if assumptions (6) and (7) are used, then it will be necessary to give careful consideration to the calculation of δ and $\bar{M}^*$ for any given process.

If the Bernoulli equation,

$$\int dp/\rho+v^2/2g=\text{constant}, \tag{7}$$

is employed for ideal gases for which

$$p/\rho=gR/\bar{M}^*T, \tag{8}$$

and for adiabatic flow with

$$p(\rho)^{-\delta}=\text{constant}, \tag{9}$$

it is found that

$$v=\{[2g\delta RT_c/(\delta-1)\bar{M}^*] \times[1-(p/p_c)^{(\delta-1)/\delta}]\}^{\frac{1}{2}}, \tag{10}$$

where p = mean hydrostatic pressure; ρ = density of the fluid mixture comprising the products of combustion; v = linear flow velocity at any point; and R = molar gas constant. Since δ generally does not vary greatly from one propellant system to another, Eq. (10) shows that for fixed values of p_e and p_c, v_e is a function primarily of $(T_c/\bar{M}^*)^{\frac{1}{2}}$. Furthermore, by Eq. (5b), the *Isp* is proportional to v_e. Hence it follows that maximum *Isp* will generally be obtained for propellant systems for which the ratio $(T_c/\bar{M}^*)^{\frac{1}{2}}$ is large. This fact has been used extensively in discussing various aspects of chemical propellants.

For adiabatic expansion the exit temperature T_e can be expressed in terms of the chamber temperature T_c, the pressure ratio p_c/p_e, and the value of δ according to the relation

$$T_e=T_c(p_e/p_c)^{(\delta-1)/\delta}. \tag{11}$$

In addition to the preceding relations, several other useful parameters can be calculated for rocket engines. The usual procedure for obtaining the desired results involves the use of assumptions (6) and (7). We shall not reproduce these developments here. Instead we define a *characteristic velocity* c^* by the relation

$$c^*=(1/\lambda)(RT_c/\bar{M}^*)^{\frac{1}{2}} \tag{12}$$

and a *nozzle thrust coefficient* C_F by the relation

$$C_F=c/c^*, \tag{13}$$

where

$$\lambda=[2/(\delta+1)]^{(\delta+1)/2(\delta-1)}, \tag{14}$$

i.e.,

$$\lambda\simeq(0.1047/\delta)+0.5048 \tag{14a}$$

for the range of values of δ ordinarily encountered. In the special case where δ and $\bar{M}^*$ are independent of T it can then readily be shown that the thrust on the rocket motor is

$$F=C_Fp_cA_t, \tag{15}$$

where A_t represents the cross-sectional area at the nozzle throat.

The development of accurate techniques for calculating v_e, without assumptions (6) and (7), is discussed in Sec. II for the special case in which no chemical reactions occur during adiabatic expansion through the nozzle (flow with "frozen" chemical equilibrium). Similar calculations for expansions in which complete thermodynamic equilibrium is maintained at all times (flow with chemical equilibrium) are described in Sec. III. The two types of calculations exemplified by chemically frozen flow and by flow with chemical equilibrium yield reasonable lower and upper

limits, respectively, for the performance obtained in properly designed rocket engines. A study of kinetic processes occurring during nozzle flow may indicate the probable proximity of the flow process to frozen or equilibrium flow.[5] The evaluation of motor performance is further complicated by the possibility of occurrence of lags in adjustment of internal energy states.[5]

II. DETERMINATION OF PROPELLANT PERFORMANCE FOR FROZEN CHEMICAL EQUILIBRIUM

For frozen chemical equilibrium the average molecular weight obviously remains constant during expansion. As starting relation we shall utilize Eq. (6) in the form

$$\tfrac{1}{2}\bar{M}v_e^2=\Delta H_c^e, \tag{16}$$

where $\bar{M}$ represents the constant molecular weight of the gas mixture and ΔH_c^e is the molar enthalpy change between nozzle entrance and exit positions. For ideal gases the quantity ΔH_c^e is independent of pressure. If the relative concentrations of each component, as well as the numerical values of T_c and T_e, are known, then it is a simple matter to evaluate both $\bar{M}$ and ΔH_c^e. The value of T_e and the composition of each constituent at any given temperature, can be calculated by well-known, though laborious, procedures derived from thermodynamic considerations of chemical equilibrium.[6–9]

The average molecular weight of a gas mixture of m components is given by the relation

$$\bar{M}=\sum_{j=1}^{m} N_jM_j, \tag{17}$$

where N_j represents the mole fraction of the jth component whose molecular weight is M_j. The value of ΔH_c^e for frozen chemical equilibrium is evidently given by the relation

$$\Delta H_c^e=\sum_{j=1}^{m} N_j[H_j(T_c)-H_j(T_e)], \tag{18}$$

or

$$\Delta H_c^e=\sum_{j=1}^{m} N_j\int_{T_e}^{T_c} Cp_j dT, \tag{19}$$

where $H_j(T)$ represents the molar enthalpy of component j at the temperature T and Cp_j is the corresponding molar specific heat. Note that N_j remains constant during frozen chemical flow. Equation (18) follows immediately from the fact that the enthalpy of an ideal gas mixture is equal to the sum of the enthalpies of individual molecular species.

Since T_e and N_j are uniquely defined by the requirement that thermodynamic equilibrium is maintained in the combustion chamber,[6–9] it is only necessary to consider suitable methods for the calculation of T_e. The exit temperature is most conveniently determined by utilizing the fact that adiabatic and nonviscous flow through a nozzle is isentropic.[10] Hence

$$\bar{S}(T_c, p_c)=\bar{S}(T_e, p_e). \tag{20}$$

For ideal gases

$$\bar{S}(T, p)=\sum_{j=1}^{m} N_jS_j^0(T) - R[\ln p+\sum_{j=1}^{m} N_j \ln N_j], \tag{21}$$

where $S_j^0(T)$ represents the absolute molar entropy of pure component j at a pressure of one atmosphere and the temperature T. The quantity $S_j^0(T)$ may be obtained from entropy tables for ideal gases.[11] The exit temperature is readily determined by trial and error using Eqs. (20) and (21).

Reference to the material presented in Sec. I shows that the complete evaluation of rocket performance parameters requires knowledge of δ. Following an approximate method of perform-

[5] For a study of kinetic processes during nozzle flow, as well as for a discussion of the thermodynamics of propellant evaluation if lags in internal energy occur, see S. S. Penner, J. Chem. Phys. **19**, 877 (1951), and the references cited in this article.

[6] R. R. Wenner, *Thermochemical Calculations* (McGraw-Hill Book Company, Inc., New York, 1941), Chapter 11.

[7] S. R. Brinkley, Jr., J. Chem. Phys. **14**, 563 (1946); **15**, 107 (1947).

[8] H. J. Kandiner and S. R. Brinkley, Jr., Ind. Eng. Chem. **42**, 850 (1950).

[9] An analog computer for the determination of flame gas compositions has been described by W. S. McEwan and S. Skolnik, Rev. Sci. Instr. **22**, 125 (1951).

[10] Considering frozen flow as being isentropic flow involves the subtle recognition that a part of the phase space normally accessible to the gas mixture has become inaccessible.

[11] National Bureau of Standards, *Tables of Selected Values of Chemical Thermodynamic Properties*, Series III, Vol. I, March 31, 1947 to June 30, 1949.

ance calculations developed by Stosick,[12] we can use Eq. (11) to calculate δ once T_c and T_e have been determined for given values of p_c and p_e. For convenience the method for determining the performance parameters of rocket engines for frozen chemical flow is summarized below.

Outline of Performance Calculations for Frozen Flow

(1) Calculate the adiabatic flame temperature T_c and the equilibrium composition at T_c using standard methods.[6-9]

(2) Calculate T_e by trial and error from the relation

$$\sum_{j=1}^{m} N_j[S_j^0(T_c)-S_j^0(T_e)]=R\ln(p_c/p_e), \quad (22)$$

which is obtained from Eqs. (20) and (21) by noting that N_j is constant for adiabatic expansion without composition change.

(3) Calculate $\bar{M}=\bar{M}^*$ from Eq. (17) and the molar enthalpy change ΔH_c^e from Eq. (18) by using enthalpy tables for ideal gases.[11]

(4) Determine the value of v_e from Eq. (16).

(5) If the expansion angle α is very small and $p_0=p_c$, then Isp is given by Eq. (5b) and $c=v_e$. If α is not very small and/or $p_0\neq p_c$, then Eq. (2) must be used for the calculation of c and Eq. (5a) for the evaluation of Isp.

(6) For frozen flow δ is given, approximately, by Eq. (11) and $\bar{M}=\bar{M}^*$ whence Eq. (12) may be used to calculate c^*.

(7) The nozzle thrust coefficient C_F is given by Eq. (13) and the total thrust on the rocket engine by Eq. (15).

III. DETERMINATION OF PROPELLANT PERFORMANCE FOR EQUILIBRIUM FLOW

For equilibrium flow, as well as for frozen flow, Eq. (16) holds except that we must restrict our analysis to a *given weight* of gas mixture, rather than to the temperature-dependent molecular weight of gas. In particular, if we choose a weight equal to the initial molecular weight $\bar{M}_c$ of the gas mixture, we obtain

$$\tfrac{1}{2}\bar{M}_c v_e^2=\Delta H_c^e=\bar{H}_c-(\bar{M}_c/\bar{M}_e)\bar{H}_e, \quad (23)$$

where

$$\bar{M}_c=\sum_{j=1}^{m} N_j(T_c, p_c)M_j \quad (24)$$

and

$$\bar{M}_e=\sum_{j=1}^{m} N_j(T_e, p_e)M_j. \quad (25)$$

In order to perform numerical calculations it is convenient to consider that $\bar{H}_c$ and $\bar{H}_e$ represent the absolute molar enthalpies of the gas mixture referred to 298.16°K as reference point. Thus

$$\bar{H}_c=\sum_{j=1}^{m} N_j(T_c)[\Delta Hf_j^0+H_j(T_c)-H_j(298.16°\text{K})] \quad (26)$$

and

$$\bar{H}_e=\sum_{j=1}^{m} N_j(T_e)[\Delta Hf_j^0+H_j(T_e)-H_j(298.16°\text{K})], \quad (27)$$

where ΔHf_j^0 is the standard heat of formation of component j at 298.16°K and one atmosphere. The factor $\bar{M}_c/\bar{M}_e$ in Eq. (23) corrects the molar enthalpy at the nozzle exit position to the enthalpy for the *same weight* of gas mixture for which $\bar{H}_c$ has been computed. The enthalpy differences $\bar{H}_j(T°\text{K})-\bar{H}_j(298.16°\text{K})$, as well as the standard heats of formation ΔHf_j^0, may be obtained conveniently from the compilations prepared by the National Bureau of Standards.[11] When Eqs. (26) and (27) are introduced into Eq. (23) a term of the form

$$\sum_{j=1}^{m}[N_j(T_c, p_c)-(\bar{M}_c/\bar{M}_e)N_j(T_e, p_e)]\Delta Hf_j^0$$

is obtained which corresponds to the enthalpy change resulting from the occurrence of chemical reaction during expansion through the nozzle. This term does not appear in frozen-flow calculations because

$$N_j(T_c, p_c)=N_j(T_e, p_e)=\text{constant}$$

and

$$\bar{M}_c=\bar{M}_e=\text{constant}$$

for flow without chemical changes.

Since adiabatic expansion of ideal gases with complete chemical equilibrium is also isentropic, we can develop a relation similar to Eq. (22) for the calculation of T_e. It should again be noted

[12] A. J. Stosick, Jet Propulsion Laboratory Report No. 1-25 (California Institute of Technology, Pasadena, April, 1945).

that isentropic flow occurs for the *total weight* or any conveniently chosen *fixed weight* of gas mixture, and not for the temperature-dependent molecular weight of gas mixture. Thus, choosing $\bar{M}_e$ as the fixed weight for which the analysis is to be carried out, the occurrence of isentropic flow may be expressed by the relation

$$\bar{S}(T_c, p_c) = (\bar{M}_c/\bar{M}_e)\bar{S}(T_e, p_e). \quad (28)$$

Here $\bar{S}(T, p)$ is the molar enthalpy of an ideal gas mixture at temperature T and pressure p. It is given by an expression identical with Eq. (21) except that proper allowance must now be made for the temperature and pressure dependence of N_j.

Combining Eqs. (21) and (28) the desired result is obtained, *viz.*,

$$\sum_{j=1}^{m} N_j(T_c, p_c)[S_j^0(T_c) - R\ln p_c - R\ln N_j(T_c, p_c)] = (\bar{M}_c/\bar{M}_e)\sum_{j=1}^{m} N_j(T_e, p_e)[S_j^0(T_e) - R\ln p_e - R\ln N_j(T_e, p_e)]. \quad (29)$$

Equation (29) can be used to calculate T_e by a trial and error procedure for any given propellant system. It should be noted that the use of Eq. (29) requires numerical calculation of $N_j(T_e, p_e)$ for each assumed value of T_e. For this reason, performance calculations for equilibrium flow are much more laborious to carry out than performance calculations for frozen flow.

In order to determine, approximately, the performance parameters other than v_e, the same procedure may be used as for frozen flow. The method for calculating performance parameters for flow with chemical equilibrium is summarized below.

Outline of Performance Calculations for Flow with Chemical Equilibrium

(1) Calculate the adiabatic flame temperature T_c and the equilibrium composition at T_c, using standard methods.[6-9]

(2) Calculate T_e from the relation

$$\sum_{j=1}^{m} \{N_j(T_c, p_c)[S_j^0(T_c) - R\ln N_j(T_c, p_c)] - (\bar{M}_c/\bar{M}_e)N_j(T_e, p_e)[S_j^0(T_e) - R\ln N_j(T_e, p_e)]\} = R\ln(p_c/p_e) \quad (30)$$

by trial and error. For each assumed value of T_e it is necessary to determine the equilibrium mole fractions $N_j(T_e, p_e)$.

(3) Calculate

$$\Delta H_c^e = \bar{H}_c - (\bar{M}_c/\bar{M}_e)\bar{H}_e$$

from the relation

$$\Delta H_c^e = \sum_{j=1}^{m} \{N_j(T_c, p_c)[\Delta Hf_j^0 + H_j(T_c) - H_j(298.16)] - (\bar{M}_c/\bar{M}_e)N_j(T_e, p_e)[\Delta Hf_j^0 + H_j(T_e) - H_j(298.16)]\}. \quad (31)$$

(4) Determine the value of v_e from Eq. (23).

(5) If the expansion angle α is very small and $p_0 = p_e$, then Isp is given by Eq. (5b) and $c = v_e$. If α is not very small and/or $p_0 \neq p_e$, then Eq. (2) must be used for the calculation of c and Eq. (5a) for the evaluation of Isp.

(6) The quantity δ may be calculated from Eq. (11). Once δ has been approximated, Eq. (12) may be used to calculate, approximately, the value of c^* by setting $\bar{M}^*$ equal to $\bar{M}_c$.

(7) The nozzle thrust coefficient C_F may be determined from Eq. (13) and the thrust on the rocket engine from Eq. (15) for a given cross-sectional area of the nozzle throat A_t.

. . . the French Empire recognized Davy's work in electrolysis which had been the subject of the Bakerian lecture for 1806. While the two nations were at war, he went to Paris and received a medal which Napoleon Bonaparte had established for the Institute to award for "the best experiment which should be made in the course of each year on the galvanic fluid."—GEORGE A. FOOTE, *Isis*, **42**, 205 (1951).

The Tesla Coil*

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(Received July 15, 1951)

A description of the construction and operation of a replica of a coil, such as Nikola Tesla first described in 1891, is presented.

IF Nikola Tesla is remembered for nothing else, he will long be remembered in the academic world for the coil which bears his name. Nearly all budding scientists whose interests are along electrical lines have at one time or another undertaken the construction of a Tesla coil. The results are invariably gratifying and the apparatus is spectacular.

The Tesla coil described in this paper was made with the dual purpose of: (1) Enlarging our store of demonstration equipment and (2) making a Tesla coil as nearly like the original as possible. It has seemed to us that demonstration equipment takes on added appeal, if we can produce exact replicas of famous experimental pieces. With regard to the Tesla coil the literature would indicate that the spectacular qualities of the coil which have so endeared it to the hearts of demonstrators were by no means lost on the people of the "Gay Nineties." The coil was first described about 1891 and is referred to as a means of creating "manmade lightning." In a series of articles[1] Tesla says, "···at my lecture in 1891 (I exhibited) a coil giving sparks of *five inches*. ···Subsequent investigation showed that no matter what medium is employed, be it air, hydrogen, mercury vapor, oil or a stream of electrons, the efficiency is the same." (Italics are Tesla's.) Tesla then compares this independence of efficiency to the independence of path for a conservative force field. The comparison is not quite clear, since, while one is most certainly independent of medium and the other is independent of path in the mathematical sense, there is really no analogy beyond the fact that both have independence—of quite different kinds—in common. Further along he states, "···in 1900 I obtained powerful discharges of 100 feet."

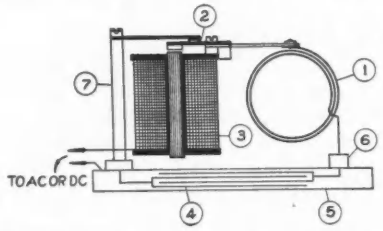

FIG. 1. Scheme of circuit connections in Tesla's oscillation transformer. (1) Primary of Tesla coil; (2) contacts; (3) spark coil with adjustable iron core; (4) condenser; (5) base mounting; (6) and (7) condenser terminals.

How great an impression Tesla's experiments made can be gathered from the amount of material which was published concerning his work. And the focal point of much of this material was his coil. The *Century Illustrated Monthly Magazine* devoted several lengthy ar-articles to Tesla's work.[2] Not only did the magazines find his material of considerable interest to their readers, but at the same time Tesla attracted a showcase full of famous contemporaries who were more than willing to be part of his demonstrations. For instance, in reference 2 we find Samuel Clemens (Mark Twain) the demonstrator in several experiments. Others who were equally prominent in their time, if not so well known today, also seemed very happy to assist in Tesla's demonstrations. In general, most of Tesla's experiments were as unusual and as simple as the inventor's own personality.

A good illustration of this point is shown in the schematic diagram of the circuit connections given in reference 1 and reproduced here as

* The authors wish to express their appreciation to John J. O'Neill, Science Editor of the *New York Herald-Tribune* and biographer of Nikola Tesla (*The Prodigal Genius*, Ives-Washburn, New York, 1944), for making available his extensive bibliography and reference works on Nikola Tesla.

[1] Nikola Tesla, Electrical Experimenter (June, 1919). This is one of a series of five articles entitled "My Inventions" which ran in the Electrical Experimenter from February, 1919, to June, 1919.

[2] T. C. Martin, "Tesla's oscillator and other inventions," Century Magazine **49**, No. 6 (April, 1895).

Fig. 1. It should be noted that Tesla does not show the secondary, or Tesla coil in his diagram, but he notes that the secondary "slips" inside the primary. This is precisely what we did in our coil; the secondary is the conical coil which has been placed inside the brass spiral which forms the primary (see Fig. 2).

CONSTRUCTION OF THE COIL

The exact method of construction which Tesla employed and the materials used are not known, nor are the exact design figures. However, from the material referenced here, it can be safely said that our coil is as nearly an exact replica as anyone could hope to make. We have used some of our own methods to prevent arcing between the primary and secondary, but beyond such trivialities we have not venture to make any improvisations.

The form for the conical coil is made of wood strips and drawing paper which has been shellacked with about ten coats of shellac. The diameter of the base of the cone is 12 inches, and the diameter of the top of the frustum is 4 inches. The vertical height of the coil is 15 inches. Plywood or any handy workable material can be used for the top and bottom of the cone. As noted previously the frustum of the cone forms the secondary, or Tesla coil, of the system. It is wound with 450 turns of No. 24 cotton-covered, copper wire in a close-spaced arrangement. The primary, which surrounds the secondary at the base, is made from No. 26 gauge half-hard brass strip 1-inch wide. The brass strip forms a spiral of four turns with an inside diameter of 15 inches and an outside diameter of about 16 inches. The strip is insulated by means of intervening layers of corrugated paper which has been dipped in paraffin.

The primary coil was found to arc with the secondary after repeated operation or during continuous operation that lasted five minutes or longer. In order to prevent this the primary was removed, redipped in paraffin and then wrapped in insulating tape. It has also been found advisable to give periodic coats of shellac to the secondary for still better insulation, since it is entirely possible for the insulation to be burned off the secondary during arcing, which would mean rewinding the entire coil. These precautions have just about eliminated arcing.

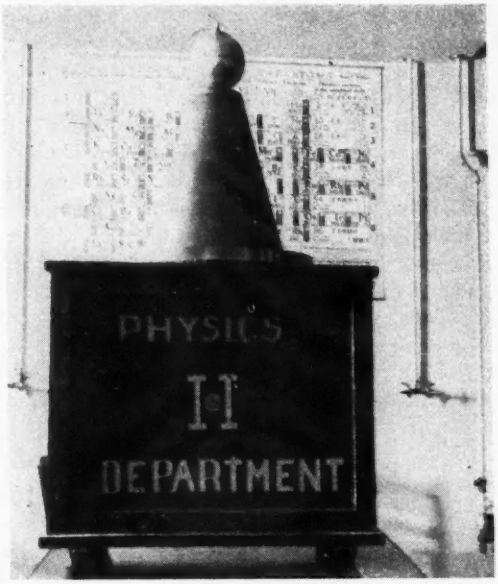

FIG. 2. Hofstra Tesla coil.

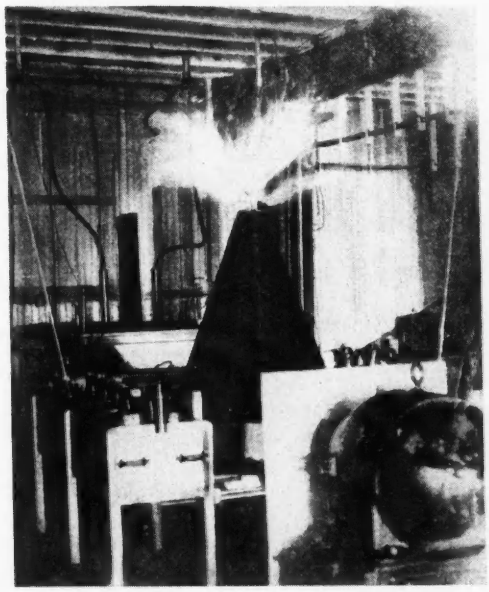

FIG. 3. Tesla's original coil, reproduced from Century Magazine **49**, No. 6 (April, 1895).

The power source consists of a 20,000-v, 20-ma transformer, a 0.01-μf, 24,000-v condenser, and a spark gap. Actually we used four banks of 6000-v, 0.01-μf condensers hooked up in series. A simpler arrangement, and one which would last longer, is to use glass and mica condensers. We are now making this substitution.

The cost of the entire unit is very small, and it makes both a spectacular and long-wearing piece of demonstration equipment, offering to students (and teachers!) the opportunity to observe what was being done in "modern" physics over fifty years ago.

FIG. 4. Hofstra Tesla coil with the "pinwheel" attachment.

In order to make it easier to move the coil, a box was made and mounted on rubber wheels. The transformer, the condenser, and the spark gap were placed inside the box. Thus all of the light flash and some of the noise from the spark gap were eliminated.

Figure 2 shows the Hofstra Tesla coil, while Fig. 3 shows the original Tesla coil[2] while in operation. Figure 3 is a reproduction of Fig. 15 from reference 2. The similarity of construction can be seen quite clearly. It might be worth noting that most of Tesla's coils were somewhat larger than ours, but we felt that a much larger one only increased the problems of construction without increasing the value, particularly for demonstration purposes.

An interesting sidelight on the size of our coil is that even in this size it is quite capable of causing considerable interference with both radio and television over a considerable distance. Despite concrete walls and floors, radios located as much as 100 feet away were rendered useless while the coil was in operation. The same thing was true of television reception. The frequency of the oscillation varies over a very wide range —from the foregoing it can be seen that it varies from the kilocycle to the megacycle range—and therefore some care must be taken in using the coil. It is also capable of giving severe burns to a careless operator, although the sensation caused by the electrical discharge is relatively mild.

DEMONSTRATIONS

A wide variety of demonstrations may be used. A few of those we found interesting are listed. Figures 2 and 4 show a ball mounted on top of the coil. The ball we used was aluminum, about 4 inches in diameter. The emanations in this experiment are in the form of long (5 to 8 inches) violet threads which dance over the surface of the ball. One experiment or demonstration which we found good was to have the demonstrator hold a 48-inch fluorescent lamp about 6 inches from the ball, or as close as was necessary in order to have a spark jump to the tube and remain in contact. Then the demonstrator held another 48-inch tube in his other hand. This second tube would, of course, light up. The demonstrator can show rather vividly and convincingly that he is actually part of an electrical circuit by moving the first tube—the one in contact with the spark—away from the spark so that contact is broken. The second tube will go out. By moving the first tube back and forth in and out of contact the second tube lights up and goes, as often as the first tube is moved.

Another demonstration we used showed very effectively the energy of the oscillations. By mounting a pin in a hole on the top of the ball and placing on this pin a pinwheel made of ordinary copper wire with the axis of rotation in the vertical plane and the rotation itself in the hori-

zontal plane, the effect shown in Fig. 4 was obtained. Immediately above the Tesla coil is a fluorescent ceiling fixture at a distance of approximately 3 feet. It can be seen that this has been quite well lighted and at the end nearest the coil was giving off light comparable to its regular power. The circular effect at the top of the coil is caused by the high speed rotation of the pinwheel. We have clocked the speed of this rotation to be at an average of 60 revolutions per minute. There is no need to list further demonstrations since the number and variety are a direct function of the demonstrator's ingenuity.

Figure 1 was made by Robert Gilliom, one of our senior students; Figure 3 was made by Robert Meyer, also one of our senior students.

Statistical Fluctuation in Radioactive Phenomena

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(Received July 19, 1951)

An experiment applicable to an intermediate laboratory course in modern physics is presented. The method is fundamentally that used in 1910 by Rutherford and Geiger, with a G-M scaler and tape recorder replacing the scintillation technique. The experiment establishes the random nature of radioactive decay and introduces the student to the concept of statistical fluctuation.

IT has been shown experimentally that the average rate of disintegration of a radioactive element is at any instant proportional to the number N of atoms present at that instant. This law may be expressed

$$dN/dt = -\lambda N,$$

the solution of which leads to the familiar exponential law of decay

$$N_t = N_0 e^{-\lambda t}, \tag{1}$$

where $N_{(t)}$ is the number of atoms present at time t, N_0 the number of atoms originally present at $t=0$, and λ the disintegration constant.

It follows from the previous law that on the average $N\lambda dt$ atoms will disintegrate in time dt. The expression λdt may be interpreted as the probability that a disintegration will occur during an interval of time dt. Now if the radioactive disintegrations are random, the fluctuations about the average value will obey statistical laws.

These statistically derived theories of decay have been checked by the experiments of Kohlrausch,[1] Meyer and Regener,[2] and Geiger.[3] Marsden and Barratt,[4] along with Mme. Curie,[5] measured the time between successive disintegrations and also verified the statistical nature of these radioactive transformations. In 1910, Rutherford and Geiger,[6] using a scintillation method and noting the variation in the number of alpha-particles given off by polonium in 8-sec intervals, did the work which, in principle, forms the basis of the present experiment. In our experiment beta-particles emitted by C^{14} were automatically recorded by means of a scaler and tape recorder.

The probability that n disintegrations will occur in a time dt is given by Poisson's equation

$$W(n) = e^{-x} x^n / n!, \tag{2}$$

where x is the average number of disintegrations which occur during the time dt.

Since in a commercial counter the mechanical register is activated by the output of a scaling circuit, each disintegration observed does not produce a count on the register. On the scale of two, the first pulse ignites the first interpolation lamp; the second pulse activates the register

[1] Kohlrausch, Wien. Ber. **115**, 673 (1906).
[2] Meyer and Regener, Ann. Physik **25**, 757 (1908).
[3] Geiger, Phil. Mag. **15**, 539 (1908).
[4] Marsden and Barratt, Proc. Phys. Soc. (London) **23**, 367 (1911); **24**, 50 (1911).
[5] Mme. Curie, J. phys. et radium **1**, 12 (1920).
[6] Rutherford, Geiger, and Bateman, Phil. Mag. **20**, 698 (1910).

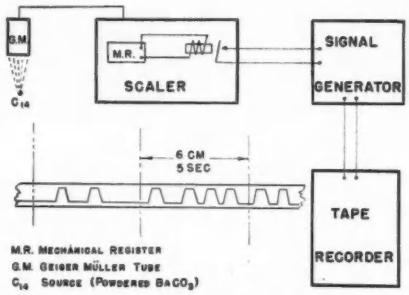

FIG. 1. Schematic diagram of the experimental setup. The G-M tube used was a mica-end window of thickness 3 mg/cm². A relay in series with the mechanical register of the scaler keyed a code-practice oscillator which actuated the tape recorder. The pip width varied from about 0.05 sec to 0.1 sec.

and records one count. Thus, the scaler records the first disintegration observed as zero counts, the second and third as one count, the fourth and fifth as two counts, etc., so that the nth count represents either $2n$ or $2n+1$ disintegrations. It is evident then that if n is the number of counts recorded by the counting mechanism of the scaler and consequently on the tape in the time dt, it is the result of the probability that either $2n$ or $2n+1$ particles were detected. Since these events are mutually exclusive, the probability that n counts are recorded by the register in the time dt is the sum of the probabilities that $2n$ and $(2n+1)$ disintegrations have been observed and

$$W(n)=W(2n)+W(2n+1)$$

$$W(n)=\frac{x^{2n}}{(2n)!}e^{-x}+\frac{x^{2n+1}}{(2n+1)!}e^{-x}$$

$$W(n)=\left[\frac{x^{2n}e^{-x}}{(2n)!}\right]\left(1+\frac{x}{2n+1}\right). \qquad (3)$$

In general, if r is the scaling factor it can be shown that the probability for n counts will be given by

$$W(n)=\sum_{i=1}^{r} W[(n+1)r-i]. \qquad (4)$$

In verifying this relationship for a scaling factor $r=2$, C^{14} was used as a source. It was in the form of powdered barium carbonate[7] and was suspended in a light grease. The beta-particles emitted were counted by means of a Geiger tube and a scale of two which records the passage of particles through the tube on a mechanical register. A relay mounted inside the scaler was connected in series with the solenoid of the mechanical register. The contacts of this relay were connected so as to key an audio oscillator, the output of which actuated a Boehme radio-telegraph recorder, which operates on 110–120 volts, 60-cycle ac. It can be activated by a radio receiver or any other source of audio frequency dot and dash signals, and responds to a maximum speed of approximately 200 words/min, or 50 pips/sec. The ink recorder consists of a permanent magnetic field with a signal coil hinged to a pen arm which records the signals on a constant speed tape. Thus alternate particles through the Geiger tube are automatically recorded as tiny pips on a paper tape moving at a constant rate. A schematic diagram of the apparatus is shown in Fig. 1.

The aforementioned method of recording counts may be simplified if an automatic record of the data is not desired. The counter may be started and stopped for any desired time interval and the data obtained directly by counting interpolation lights.

Since we are interested here only in the distribution in time, the geometry of the setup is not particularly important. The most desirable counting rate depends upon the sensitivity of

[7] The radioactive $BaCO_3$ can be obtained from the AEC, Oak Ridge, Tennessee.

TABLE I. Comparison of observed and theoretical data.

n Number of pips in an interval	N Number of intervals containing n pips			Nn
	Experimental	Theoretical $W(n)$	Theoretical N	Experimental
0	2	.005	6	0
1	33	.062	72	33
2	182	.197	228	364
3	333	.289	334	999
4	318	.245	284	1272
5	194	.134	155	970
6	70	.051	59	420
7	19	.014	17	133
8	4	.003	3	32
	$1155=\Sigma N$ Total intervals counted	$\sum_0^8 (Wn)=1.00$	1158 Total intervals calculated	4223 Total number of pips

$x=7.312$ particles per interval,

$d=x/2=3.656$ pips per interval.

the apparatus—particularly the relay and the tape recorder. It was found that best results were obtained with between 60 and 70 pips per min. The conditions under which the run to be analyzed was made are tabulated as follows:

Scalar setting	2 particles/pip
Tape speed	72 cm/min
Distance to source	10.4 cm
Length of run	1 hour 40 min
Total pips	4223 pips.

The tape obtained above was run through a template with equal intervals marked off and the number of pips in each interval noted.

Interpretation of Results

The tape obtained as described above was analyzed using 6-cm or 5-sec intervals. The results of this determination are shown in Table I, together with the theoretically computed values.

The calculated values are obtained by means of Eq. (3). Since x has been defined as the average number of pips per interval, referring to Table I we have:

$$x=\frac{\text{total number of particles observed}}{\text{total number of intervals}},$$

$$x=2\frac{\text{total number of pips}}{\text{total number of intervals}}=2\frac{\sum N_n}{N}=7.312.$$

The term n is given in pips per interval; hence a factor 2 must be introduced in order to express the total number of particles emitted.

The calculation of the expected number of intervals containing three pips in Table I is given below as an example:

$$W(n)=\frac{x^{2n}e^{-x}}{2n!}\left(1+\frac{x}{2n+1}\right),$$

$$W(3)=\left[\frac{(7.312)^{2\cdot 3}e^{-7.312}}{(2\cdot 3)!}\right]\left[1+\frac{7.312}{(2\cdot 3+1)}\right],$$

$$W(3)=0.2895,$$

$$N=W(3)\sum N,$$

$$=(0.2895)(1155),$$

$N=334$ intervals containing 3 counts.

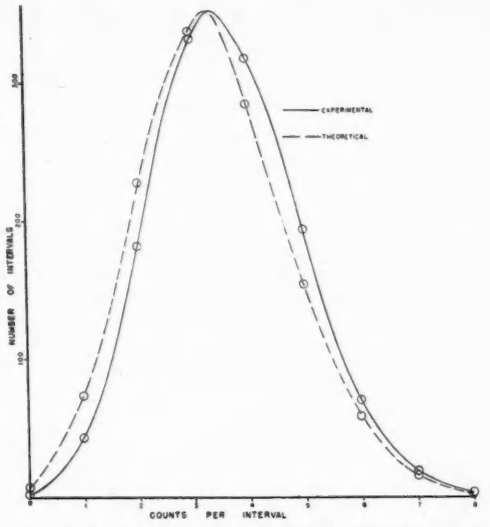

FIG. 2. Plot of data from Table I. The Poisson distribution is a probability distribution which applies where the average unscaled counting rate is very small and the number of intervals observed is large. Closer agreement between experimental and theoretical curves could probably be obtained with an average counting rate of 5 counts/interval or 2.5 pips/interval.

The number of such intervals observed was 333.

It is interesting to note that the theoretical curve is the same shape as the experimental, but displaced slightly toward the y-axis (Fig. 2). The same curve indicates that approximately one thousand intervals are sufficient to verify randomness as a student experiment. In order to obtain best results, it is recommended that the distribution of n be kept roughly between 0 and 7 pips per interval in order to simplify the counting procedure and probability calculations. Also, in this connection, it should be remembered that in order to approximate the Poisson distribution, the average counting rate should be kept small.

The experiment has been performed satisfactorily using background counting rate as the source. However, with a background of about 30 counts/min, the intervals must be longer and the experiment is appreciably lengthened.

Space at Mid-Century

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(Received June 28, 1951)

The concepts of space which are used in contemporary physics are discussed, and are contrasted with the older concepts associated with Newtonian physics. The effects of uniform motion on space and the simplest currently accepted relations between space and time are stated. Deviations from three-dimensional orthogonal space as demanded by the general theory of relativity are discussed, with attention to the relative roles of space and time terms in the space-time curvature associated with gravitational fields. The development of cosmological theory, on the basis of general relativity concepts, is briefly indicated. Mach's suggestion for the relation of matter and space is described; more recent speculations relating over-all space and local properties, and the possibility of discontinuity in microscopic space, are noted. The emphasis in the current conception of space on interdependence between space and matter is pointed out.

THE ideas which are most fundamental in our lives are often least likely to be explicitly discussed and criticized. The concept of space seems to belong to such a group of fundamental ideas. With the possible exception of temporal order, the requirement of existence in space is the most general condition for an entity's being part of physical nature. Introductory physics textbooks, however, rarely discuss the space that is basic to the physical world being studied; rather, there seems to be a tacit assumption that teachers and students well know what is referred to whenever the term "space" is used.

In most instances a silence about "What is space?" is perhaps justified, since reference is probably being made to everyday properties of space that are familiar to a child. However, as we all know, there are many aspect of physics which require concepts of space that are not apparent in ordinary experience, and further, these concepts are in considerable contrast to the generally accepted ideas about space of a few decades ago. In this paper we shall attempt to delineate some of the most important aspects of the concept of physical space that is operative in physics today. Our primary concern will be with ideas about space which, implicitly at least, might make a difference in our teaching of general introductory physics courses. In our treatment we shall assume that the special theory of relativity is correct, and that the general theory of relativity, insofar as it relates gravitational forces to a space-time structure, is to be taken as an accepted part of physics. Various further speculations concerning space will be discussed as indications of tendencies in current research.

The term "space" is freely used by both physicists and mathematicians for a wide variety of coordinate systems. Physicists, for example, introduce momentum space with any desired number of coordinates or dimensions. Mathematicians have devised spaces ranging from those with one to those with an infinite number of dimensions, and the dimensions themselves, as well as such properties as the distance between two points in a space, have been fashioned in many different ways. It has even been instructive to investigate spaces for which no definition of distance is given. We are interested, however, in the space of the physical world, and we assume that among the various kinds of space which mathematicians can hypothetically construct there are only certain coordinate sets which accurately describe the world of physical events. For physicists the space problem is, of course, just the problem of determining which are the descriptions, among the many geometric possibilities, that are in accord with physical observation and theory.

EXTENSION AND DIMENSIONS

The primary quality of space is its extension: a physical object is not at a single point, but occupies a number of different space points; also, physical objects are separated in space; in other words, there are space distances between them.

The spatial quality of extension is expressed by saying that we can set up spatial coordinate systems and measure the extent of objects, and distances between objects, in terms of the coordinates.

The coordinates which we most commonly use for spatial measurements are orthogonal Cartesian coordinates, or coordinates which have an unchanging, determined transformation relation to a Cartesian system. (We are here considering the everyday space of home and laboratory; large-scale curvature effects of space will be considered later.) The fact that space is adequately described by a triply orthogonal set of coordinates is, after the fundamental quality of extension, perhaps the most striking characteristic of space. In common language, this triple orthogonality is expressed in a statement such as "objects have length, width, and breadth." In physics we work with x, y, and z components of vectors; a displacement or other vector quantity in the x direction does not involve a y or z displacement or quantity. This would not be so if space were not triply orthogonal; thus in a curved space, which is not triply orthogonal, motion along one of the Cartesian coordinate axes might in certain cases give additional Cartesian components of motion.

The independence of vector quantities in one dimension of space from quantities in the other dimensions is a consequence of the orthogonality of the dimensions; or indeed, one could say that this independence is the orthogonality property. The fact that there are three such dimensions, and not two or four or some other number, is apparent in common sense considerations of "length, width, and breadth." In physics we find that there are just three degrees of translational freedom. A common illustration of three-dimensionality is seen in the stability of a stool with three legs on any surface. In contrast, a two- or four-leg stool is stable only under highly specialized conditions. The milkmaid's stool is a good argument against those who would see three-dimensionality as only a convention to which we have become habituated.[1]

[1] The speculative physicist who wishes to ask "Why are there three dimensions?" will find a discussion of this question on pp. 135–137 of Herman Weyl's *Philosophy of Mathematics and Natural Science* (Princeton University Press, Princeton, New Jersey, 1949).

RELATIONS OF SPACE AND TIME

Nothing that we have said so far would be questioned by a nineteenth-century physicist. The most important single discovery that has been made about space in the twentieth century is undoubtedly the relatedness of space and time. This discovery was effectively made by Einstein, and reported in his first relativity paper in 1905.[2] The relation of space to time is required by special relativity theory, in that the interval which remains the same in a transformation from one coordinate system to another moving uniformly with respect to the first is not an interval of spatial dimensions only, or of the time dimension only, but rather is composed of both and space and time intervals. This formal requirement leads to explicit relations between space and time intervals in the special relativity equations. Thus, if we consider, in the usual relativity theory manner, a coordinate system S and a second system S^* moving at a constant velocity v in the x direction with respect to S, we have the relativistic transformation[3] between a time interval δt^* in S^* and space and time intervals δx and δt of S

$$\delta t^* = (\delta t - v\delta x/c^2)/(1-v^2/c^2)^{\frac{1}{2}}. \qquad (1)$$

Two events at different space points in the moving system S^* may be simultaneous in that system, so that $\delta t^* = 0$. Equation (1) then states that clocks in the system S will find the events to have a time separation $\delta t = v\delta x/c^2$, where δx is the spatial separation of the events to the S observer. Hence, the space interval δx^* in S^*, which has no time interval associated with itself, does imply a time interval for the S observer; or, to put it another way, physical events which are simultaneous to a rest observer will not be so to a moving observer. Inasmuch as the time interval between the events is proportional to the spatial separation x, interdependence between space and time is here displayed.

As we expect in relativity theory relations, if v is zero or very small relative to the velocity

[2] Ann. Physik, Ser. 4, 17, 891 (1905). English translation by W. Perrett and G. B. Jeffery in *The Principle of Relativity* (Dodd, Mead, and Company, New York, 1923).

[3] See, for example, R. C. Tolman, *Relativity, Thermodynamics, and Cosmology* (Oxford University Press, London, 1934), p. 22, or any other exposition of special relativity theory.

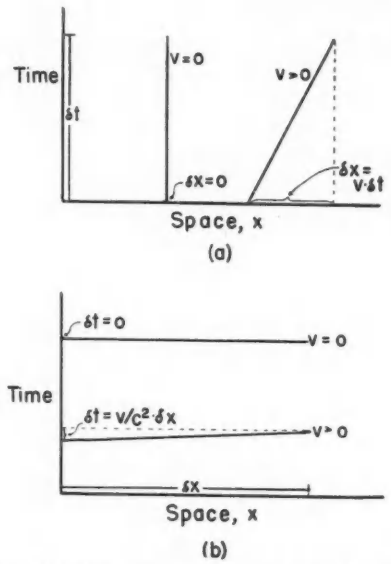

FIG. 1. Relations between space and time intervals are shown for the velocity v of S^* equal to zero and for v greater than zero. (a) A particle at rest in S^* moves through a distance $v\delta t$ in time δt. Or, for a particle moving with a velocity v, a space interval $v\delta t$ is necessarily associated with a time interval δt. (b) A space interval, moving with velocity v and measured as δx in the observer's rest system, implies a time interval $\delta t = v\delta x/c^2$. Hence, a time interval is necessarily associated with a moving space interval.

of light c, we obtain a zero or negligible time interval δt in association with the space interval δx. In general, we find that as long as we are considering a physical situation that does not involve high relative velocities, we can take space and time to be independent, as in pre-relativistic physics; but, of course, a physics so limited describes only a small part of physical phenomena.

The interdependence of space and time intervals has some foreshadowing in the equations of classical Newtonian physics (although no such relation is contained in the explicit assumptions about space and time used by Newton). Thus, as we teach our freshman students, any body moving through an interval δx with a velocity v requires a time $\delta t = \delta x/v$ for its motion. This relation is given in relativity theory by the analog of Eq. (1). The analogous transformation equation for a space interval δx^* is

$$\delta x^* = (\delta x - v\delta t)/(1 - v^2/c^2)^{\frac{1}{2}}. \qquad (2)$$

The moving particle which is observed to be at rest in S^* (i.e., the velocity v of the particle is the velocity of S^*) will remain at a constant x^* value and we have $\delta x^* = 0$; hence, by Eq. (2), $\delta t = \delta x/v$, as in classical mechanics. In other words, classical mechanics gave us a space interval associated with the continuous motion of a body throughout a given time interval; relativistic physics gives us this, but in addition it gives a time interval associated with a moving space interval. Expressed in another way: classically, a time interval is required for the passage of a point with velocity v through some space interval; relativistically, the mere motion of a space interval with velocity v requires that there be a time interval between points that are spatially separated but at rest with respect to each other. The factors which give the time intervals, when multiplied by the space intervals in question, are $1/v$ and v/c^2 for the classical and relativistic cases, respectively—hence the relative obscurity in mechanics of the time associated with a moving space interval. (See Figs. 1(a) and 1(b).)

EFFECTS OF MOTION ON SPACE

Having noted the interdependence of space and time, we shall in the remainder of this paper discuss space in isolation from the composite physical entity, space-time. There may be objections to this procedure. There are the justifications, however, that space is psychologically different from time, and also that space and time variables, in spite of their inter-relatedness, are always mathematically distinct in physical theory.

In Newtonian physics spatial quantities are independent of motion. In relativity physics, as is well known, the transformation equation, Eq. (2), for the space variable in the direction of motion leads to a contraction of lengths measured along that direction. Specifically, an observer in S will find a length, which is δx^* to an observer in S^*, to have the length

$$\delta x = (1 - v^2/c^2)^{\frac{1}{2}}\delta x^*.$$

Space, then, is not unaffected by motion; rather, space is contracted along an observer's line of motion in accordance with the preceding relation.

It may seem to the reader that it is an inde-

fensible step to conclude from the contraction of measured lengths that space is thereby contracted. The contrary view would be that space is an absolute, unchanging entity that is apart from the objects in space, and that we never measure a spatial extension, but always the extension of some object in space or the distance between two objects in space. We are here taking the point of view, however, that spatial properties are determined by measured relations of physical objects. This assumption is in the spirit of today's physical and philosophical thought.

The transformation equations of special relativity theory give not only a contraction of space, for a moving as compared with a chosen rest observer, but also a distortion of space, in the sense that angles between lines take different values for a moving than for a rest observer. This distortion can be readily seen by considering S and S^* coordinate systems, where the velocity of S^* with respect to S is now not entirely along the x axis, but also has a y component. The transformation equation, Eq. (2), then becomes

$$\delta x^* = (\delta x/v^2)[v_x^2/A + v_y^2] + v_x v_y(1/A - 1)(\delta y/v^2) - (v_x/A)\delta t, \quad (3)$$

where A is $(1-v^2/c^2)^{\frac{1}{2}}$, v_x and v_y are the velocities of S^* with respect to S in the x and y directions, and v is the total velocity. There is a similar expression for δy^*.[4] If we consider measurements to be made by an observer in S of a line in S^*, of length δx^* with $\delta y^* = 0$, the measured line will, to the S observer, have a δy component in general not equal to zero. In other words, the line which in S^* is parallel to the x^* axis will to the S observer make an angle ϕ with the x axis, even though x and x^* are parallel when v is zero. Specifically, ϕ has a value given by

$$\tan\phi = (A-1)/(v_y/v_x + v_x A/v_y).$$

The distortion can perhaps be more simply illustrated by returning to the usual case of S^* with a velocity only along the x axis with respect to S, and considering a square in S^* with a diagonal along the x^* axis. (See Fig. 2.) To the S observer the x diagonal is contracted, but the diagonal perpendicular to x is unchanged in length; hence the square becomes a parallelogram with angles different from 90°, and the angles between sides of the square and the x axis are changed.

The effect of motion on spatial dimensions has not been explicitly observed as have such other immediate consequences of the theory of relativity as the decreased tempo of atomic clocks with motion and the mass-energy equivalence. Nonetheless, the special relativity theory is too well confirmed to allow us arbitrarily to reject any consequences of it. If we accept the theory, it is no longer permissible to think of space in the absolute sense of Newtonian physics. Rather, we must think of spatial dimensions as having a sort of "shrinkability" characteristic. In the world of velocities small compared to the velocity of light in which we have our immediate sense experience, that quality is not apparent; but at high relative velocities space loses its seeming indifference to motion.

SPACE CURVATURE

It should be emphasized that the shrinking and distortion properties of space as just discussed are in no way related to the "curvature of space," as that expression is used in physics. No space curvature is involved in relative uniform motion, for even though spatial dimensions in S^* are distorted as compared with dimensions

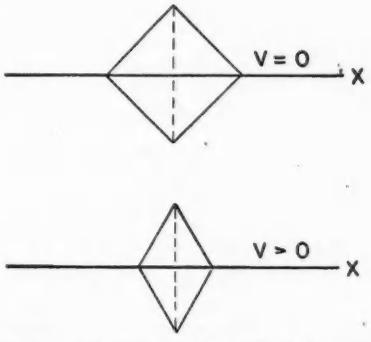

FIG. 2. A velocity in the x direction with respect to the observer gives a contraction of the moving system along the x axis, but no effect perpendicular to the x axis. Hence the 90° angles of the square are increased or decreased as v increases.

[4] These transformation equations are given in E. Madelung, *Die Mathematischen Hilfsmittel des Physikers* (Dover Publications, New York, 1943), p. 272. They may be found by an extension of the methods customarily used for finding the transformation equations for motion in one dimension.

in S, the space of S^* could always be properly described by the S observer in terms of, say, a Cartesian coordinate system. Space curvature was introduced into physics with Einstein's general theory of relativity, which in its present state of development is essentially a theory relating space-time curvature and gravitational effects. The space curvature associated with gravitational forces as we know them is actually extremely small—trivial in most cases of practical interest. The form of the general relativity equation for the problem of a central gravitating force will show the magnitude of the curvature, as we shall now see.

In special relativity theory, where no deviations from "flat" Euclidean space are indicated, the space-time interval ds between two events is defined by

$$(ds)^2=c^2(dt)^2-(dx)^2-(dy)^2-(dz)^2,$$

where dt is the time interval between the events, c is as usual the velocity of light, and dx, dy, dz are space intervals between the events. We can equally well use spherical coordinates, r, θ, ϕ, for the space terms; then

$$(ds)^2=c^2(dt)^2-(dr)^2-r^2(d\theta)^2-r^2\sin^2\theta(d\phi)^2.$$

The space-time interval is analogous to the Euclidean space interval dl, which by Pythagoras' theorem is given by

$$(dl)^2=(dx)^2+(dy)^2+(dz)^2;$$

just as the length dl is unchanged by motion in classical geometry, so is the space-time interval ds unchanged (but not the space or time intervals alone) in transformations from a coordinate system S to a system S^* that is moving uniformly relative to S.

Now, the general relativity theory tells us that if we are in the neighborhood of a mass, and wish to consider gravitational effects of the mass, the space-time interval ds is not "flat" as defined in the special theory. Instead, there is a space-time curvature which results from the presence of the mass. This curvature is expressed in the new form of the space-time interval. If the mass m is at the center of a coordinate system r, θ, ϕ, the interval is given by

$$(ds)^2=(1-2km/r)c^2(dt)^2-(dr)^2/(1-2km/r)-r^2\sin^2\theta(d\phi)^2-r^2(d\theta)^2, \quad (4)$$

where k is related to the Newtonian gravitational constant, G, by the relation, $k=G/c^2$ ($G=6.664\times10^{-8}$ dyne cm^2 g^{-2}). The variables r, θ, ϕ, t in Eq. (4) are not precisely the same as those variables in Euclidean space, for they are coordinates in a different space. But they may be considered to be expressions of radial distance, angular displacements, and time variable, and hence as analogous to the r, θ, ϕ, t of conventional laboratory physics.[5]

We consider the simple case of a unit test mass moving under the gravitational influence of a mass m. We can then see what terms in Eq. (4) for the space-time structure near a mass m give rise to ordinary Newtonian gravitational effects. To do this, we arbitrarily rewrite Eq. (4) with a coefficient of unity, instead of $1/(1-2km/r)$, for $(dr)^2$. The resulting equation differs from the equation for the interval of "flat" space-time in the $(dt)^2$ term only:

$$(ds)^2=(1-2km/r)c^2(dt)^2-(dr)^2-r^2\sin^2\theta(d\phi)^2-r^2(d\theta)^2. \quad (4a)$$

Equation (4a), which expresses no curvature for the space differentials, will give the same relation between space and time differentials as is found for the differentials along the path of a unit test mass that is moving in a central force field in accordance with Newton's law of gravity. The following development shows this property of Eq. (4a): If the unit mass has a velocity that is small compared with c, as would be the case in Newtonian considerations, the space interval traveled by the mass in time dt is small compared with cdt. Hence, we can consider $ds\simeq cdt$. If we use this approximation, division of Eq. (4a) by $c^2(dt)^2$ gives

$$1=1-2km/r-(dr/dt)^2/c^2-r^2\sin^2\theta(d\phi/dt)^2/c^2-r^2(d\theta/dt)^2/c^2,$$

or,

$$2kmc^2/r=-\dot{r}^2-r^2\sin^2\theta\,\dot{\phi}^2-r^2\dot{\theta}^2. \quad (5)$$

Since motion under any kind of central force may be taken in a plane, the term in ϕ in Eq. (5) is zero. Differentiation of Eq. (5) with respect to time then gives

$$kmc^2/r^2=\ddot{r}+r\dot{\theta}^2+r^2\dot{\theta}\ddot{\theta}/\dot{r}. \quad (6)$$

[5] Equation (4) is the so-called Schwarzschild line element, a solution of the general relativity field equations for the case of a central attracting mass, given by K. Schwarzschild in 1916.

For motion in any kind of central force[6] field, $2\dot{r}\dot{\theta}=-r\ddot{\theta}$; hence, Eq. (6) may be reduced to $kmc^2/r^2=\ddot{r}-r\dot{\theta}^2$. Or, using $k=G/c^2$,

$$Gm/r^2=\ddot{r}-r\dot{\theta}^2. \tag{7}$$

The two terms on the right are the usual acceleration terms for motion under a central force;[7] the term on the left is the usual Newtonian expression for the gravitational force on a unit mass distant r from m.

The significance of Eq. (7) is that it follows from Eq. (4a), which is the relativistic equation for space-time structure in the neighborhood of a mass m, modified so as to include a variation from "flat" space-time in the time term only. As far as its spatial terms go, Eq. (4a)—unlike Eq. (4)—presents no change from Euclidean space. So we see that even in general relativity theory, for velocities small compared with c, the effects described by Newton's law of gravity require no change from Euclidean space in the space structure about a mass. The "curvature" required for these Newtonian effects is entirely associated with the time term in the space-time interval described by Eq. (4).

In most cases Newton's law is a highly accurate description of gravitational phenomena, and therefore we are generally justified in neglecting the space curvature that is a part of the gravitational field. However, in regions close to large masses there is a discernible effect of this curvature. The celebrated case is that of the precession of the perihelion of the orbit of Mercury, in which the complete form, Eq. (4), of the space-time interval gives agreement with observation, while a form which corresponds to Newton's law does not give agreement. (Aside from predictions of small deviations from Newton's law, which have been confirmed, there are also compelling theoretical reasons which lead to acceptance of the space-time curvature, field theory of gravitation.)

A questioning student, presented with the indication of space curvature given by the general relativity theory, may ask, "How much is space curved?" Equation (4) gives a way of answering this question. We consider a plane containing the central mass m, with ϕ a constant. Then, by Eq. (4), a space interval along the radius enters into the total interval $(ds)^2$ not as $(dr)^2$ but as $(dr)^2/(1-2km/r)$, while an interval of arc along the circle enters as $r^2(d\theta)^2$. The ratio of differential radius to differential arc length is not, then, $dr/(rd\theta)$, as in Euclidean space, but is greater by a factor $(1-2km/r)^{-\frac{1}{2}}$, where r is the distance from the mass center. At the mean distance of Mercury from the sun, 5.8×10^{12} cm, the value of $(1-2km/r)^{-\frac{1}{2}}$ is $1+2.5\times10^{-8}$. (Here of course, m is the mass of the sun.) At the surface of the earth, with r the earth's radius, the earth's mass taken as m gives the value $1+8\times10^{-10}$ for $(1-2km/r)^{-\frac{1}{2}}$, so the space curvature resulting from the earth's gravitational field is indeed small.[8]

We see from the form of Eq. (4) that there is a singularity in the space-time interval equation at mass points, where $r=0$. The general theory of relativity does not give us any information about space (or space-time) within matter. We see here the same kind of difficulty that the field descriptions of contemporary physics generally face at the regions where the fields presumably have their origins. The hope has been expressed by Einstein[9] that the recently published new general field equations will lead to solutions in which the mass and charge singularities will cancel each other out, giving a singularity-free description of the electromagnetic and gravitational fields at all points inside as well as outside matter. This hope seems not be shared by many other eminent contemporary physicists: the feeling seems rather to be that some radical change in our conception of the physical field will be required for a description of fields at their points of physical origin. This question will be touched on again near the end of this paper.

A. S. Eddington pointed out that the space curvature effect of a mass would change the

[6] This follows from the so-called law of areas, $r^2\dot{\theta}=$const, by time differentiation.

[7] See, for example, R. B. Lindsay, *Physical Mechanics* (D. Van Nostrand Company, Inc., New York, 1950), second edition, p. 71.

[8] See Tolman, reference 3, p. 208. A very clear discussion of the physical significance of space curvature is given by H. P. Robertson in his essay, "Geometry as a branch of physics," in the volume *Albert Einstein: Philosopher-Scientist* (Library of Living Philosophers, Inc., Evanston, Illinois, 1949), pp. 315–332.

[9] See his essay in *Albert Einstein: Philosopher-Scientist* (reference 8), pp. 79–81, 89–95.

value of π in the neighborhood of the mass.[10] A circle of radius r described about a mass m would have a measured circumference of $2\pi r$. Measurement of the radius intervals would imply a summation over intervals $dr(1-2km/r)^{-\frac{1}{2}}$, which would always be larger than intervals dr. Hence, the measured radius would be greater than r, and the ratio of circumference to diameter would be less than the π of Euclidean space. Eddington calculated that "if a mass of a ton were placed inside a circle of 5 yards radius, the defect in the value of π would only appear in the twenty-fourth or twenty-fifth place of decimals." Euclidean π has recently been determined

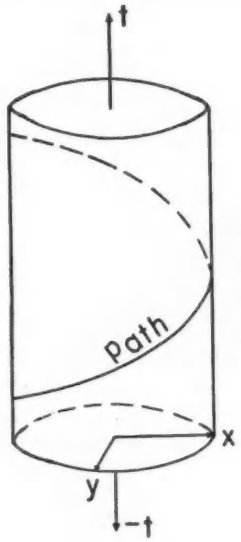

FIG. 3. The Einstein static cosmological model. The z axis of space is suppressed. The helical path represents a "journey around the universe." Motion upward along the t axis is toward the future.

to thousands of decimal places by means of high speed calculating machines; one might well raise the question of where is the physical space that would presumably fit the calculated π.

Although space curvature is slight even in the neighborhood of planetary masses, the principle of the determination of space-time structure by mass is clearly one of the fundamental new ideas of twentieth-century physics. It is an idea altogether different from the Newtonian concept of mass and space; in Newton's mechanics, bodies exist in space without in any way affecting spatial structure, and the forces which exist between bodies are not, whatever else they may be, concomitant with space structure variations.

COSMOLOGICAL SPACE

Beyond its importance as a principle in our science of nature, the utility of Einstein's space-matter relation has been largely in cosmological research. It is only in these domains that the general relativity equations have given results significantly different from results based on Newton's gravitational law. One can state the matter more emphatically: even aside from methodological objections which appear in the light of relativity theory to the simple inverse square gravitational law, that law is relatively useless in the problem of cosmology, or over-all space structure, because it gives no relation between space structure and the observed mass distribution in the universe. The general relativity theory of mass and space, on the other hand, has led to credible models of the universe which, because of the mass-space relation, can be rejected or confirmed on the basis of astronomical observations of mass density and distribution in the universe. It is not our purpose in this paper to review the cosmological models, and evidences pro and con, which have been proposed on the basis of Einstein's general mass-space-time equations.[11] But we wish to give some indication of the change in conception of cosmic space which has been associated with the cosmology of the past thirty years.

The notion is widely held that our everyday space stretches indefinitely in all directions and contains an unending number of stars and nebulae distributed everywhere in about the same density as we see in the heavens about our planet. This conception, however, is hardly a tenable one. The first and most simple objection is that we should expect the skies to be ablaze with light if space and the nebulae extended indefinitely; for, since the stellar matter would increase with the third power of any given radius, while the intensity of light from a source falls off as the inverse second power of the distance, we would always get increased light intensity as

[10] *Space, Time, and Gravitation* (Cambridge University Press, London, 1923), p. 104. The reader is referred to Chapter VI of this book for a detailed, semipopular discussion of the space-time curvature about a central mass.

[11] A complete discussion is given by Tolman, reference 3. Also, see the article by H. P. Robertson, "On the present state of relativistic cosmology," Proc. Am. Phil. Soc. **93**, 527 (1949).

we increase the space volume considered. There is the possibility of compensating absorbing materials, but astronomical evidence is at present inconclusive with respect to this hypothesis.

A second serious objection to the infinitely extended universe arises when the gravitational potential at any point is calculated on the basis of Newton's gravitational law. The potential at a point is defined as $\sum_i Gm_i/r_i$, where m_i are various masses at distances r_i from the point in question and G is the gravitational constant, as previously given. If a uniform large-scale matter density ρ is assumed (the scale must be such that unit volume contains many nebulae) the potential at a point is given by

$$\int_0^r G\rho 4\pi r^2 dr/r = G\rho \int_0^r 4\pi r dr.$$

We see then that if $r \to \infty$ the gravitational potential becomes an infinitely large, indeterminate quantity. The contrary fact, that we do observe determinable differences in gravitation potentials at different points, is a strong argument against a universe of infinite space and matter.[12] In the Newtonian cosmologies that have been constructed it is commonly assumed that matter density decreases with distance from the center of the model, although space may go on indefinitely.

The pioneer step in developing a cosmological model on the basis of the relativity equations was taken by Einstein himself, more or less concurrently with the development of the general theory of relativity. His model of the universe can be represented as a cylinder in space-time, with the time variable as the z axis, and one of the three spatial dimensions omitted (Fig. 3). The cylinder extends without limit along the time axis, but has a finite radius, which, in the theory of the model, is functionally related to the observed average density of matter. On the assumption that the density is the least indicated by astronomical observations of the space distribution of nebulae, namely 10^{-30} g/cm^3, the radius is calculated to be 3.5×10^{10} light years; higher density values would give smaller radius values. (The penetration into space by the 200-in. Palomar telescope is expected to be about 10^9 light years.)

The remarkable feature of the Einstein model is of course the finite radius; his universe is a *closed* one, finite and yet without boundaries. The analogy of the surface of a sphere is helpful in gaining some intuitive feeling for such a space; just as a two-dimensional spherical surface is finite and yet unbounded, as long as one remains within the two dimensions of the sphere's surface, so the closed three-dimensional space is unbounded; yet a light ray or particle moving in a straight line will eventually return to its starting point because the space has "curved back to the starting point." The path of a particle moving once around the universe would be represented by one turn of a helical spiral on the cylindrical surface of Fig. 3. The particle or ray cannot leave the space, for the closed, curved space constitutes all of space.

The Einstein cosmological model gave for the first time since the more or less mythological universes of Greek and medieval physics some hope of comprehending by the methods of science the cosmic structure of space. That model has been shown, however, to have unsatisfactory features in points of detail. It is, first of all, a static model, and therefore not in accord with the famous red shift, which indicates the universe to be at present in an expanding state. Second, it has been shown that the very static quality of the model is unstable; that is, the relativity principles relating space structure and mass-energy do not permit the Einstein model to retain its form under any slight internal disturbance. Other workers, following Einstein, have presented various models which are consequent upon the relativity field equations. Not only have models which account for the expansion of the universe been discovered; some of the models indicate an oscillatory behavior, with contraction and expansion alternating. Also, it has been shown that a certain arbitrary feature of the original Einstein equations—the so-called "cosmological constant"—no longer need be included in order to obtain reasonable models.

[12] This argument, as well as various cosmological possibilities based on Newtonian physics, are discussed by F. Selety, Ann. Physik **68**, 281 (1922). The existence of darkness in the sky as an argument for a finite universe is presented in detail by Olbers, *Astronomisches Jahrbuch fur 1826*, ed. J. E. Bode, pp. 110–121. An interesting discussion in favor of an infinite universe is given by Svante Arrhenius, in terms of the Newtonian physics of his day, in the *Monist*, April, 1911, p. 161.

Appealing as the closed, curved universe is, it cannot even be said that the interpretations of the observational evidence now available definitely indicate space to have this type of over-all structure. The possibility of curved, open universes, with nevertheless an effective horizon which limits the volume of universe which any one observer can know, cannot be ruled out, and models of such a universe are among those which have been developed from the general relativity equations. A new model of this general nature has recently been constructed by the English astrophysicist, F. Hoyle, and his co-workers, proceeding from a modification of the general relativity equations whereby a term that provides for the spontaneous creation of matter in space (at the slow rate of about 1 H atom per liter per 10^9 years) has been added to the equations.[13] Such a "continuous creation" idea is a startling and unlikely one to physicists, but it must be said that Hoyle and his group have been able to adduce considerable evidence in support of their hypothesis.

It is apparent that we cannot today say that any one of the cosmological models that have been proposed is clearly the best representation of the over-all structure of the universe. What we can say is that the cosmological consequences of the relativity theory now give us promising leads for understanding the over-all structure of space. And certainly we should give up any *dogmatic* belief in a flat, Euclidean space, of infinite extent and containing infinite matter; that view at best had little in its favor, and is now as an old habiliment, discarded in the development of physical thought.

LINKAGE BETWEEN LARGE AND SMALL

The fact that matter has an effect on space-time structure suggests the possibility that the properties of space are completely determined by the matter in the space. The local space of our everyday life is characterized by three-dimensional orthogonality; and matter in this space is characterized by certain inertial properties. These inertial and orthogonality properties may be determined by the large-scale distribution of matter in the universe; such was the principle proposed by Ernst Mach in the early years of this century. On this suggestion, which we shall now discuss, the universe is closely knit throughout; the behavior of a small bit of matter before you is reflecting the large-scale distribution of matter in the total universe.

The inertia of a body is clearly related to the structure of space. A body, as Newton's first law tells us, continues in its state of rest or uniform motion in a straight line unless a force acts upon it. The relativity theory states this same fact more generally by prescribing that a body move along the line of shortest extent in space-time. (The prescription is in fact a necessary consequence of the relation between matter and space-time structure, as given in the theory.) Acceleration in any direction requires that a force be applied to the body, and we give the name inertia to the resistance to acceleration; thus, in Newton's second law, inertia, or mass m, is F/a. The relatedness of inertia to space structure is shown in the fact that the inertial motion of a body is described in terms of the geometry of the space, as when we say that a free body will continue in motion in a given direction, and also by the fact that change of velocity requires a force. We see that space can hardly be like "a great vessel, indifferent to the presence of matter in it," as physicists have sometimes seemed to imply, for in that case we would not expect a force to be required for change in the direction or magnitude of velocities, nor would we expect space to constrain the motion of a body in a given direction, as it actually does.

The space-inertia relation is well illustrated in circular motion. Since a body in circular motion is continually changing its direction, an accelerating force is required to maintain the motion. It is is in connection with such motion that Mach's suggestion for the relation of matter, space, and inertia has usually been discussed. Suppose we think of the earth's rotational motion on its axis to be removed from our direct observation by a thick cloud layer, which continually obscures the sun and other heavenly bodies. We ask the question, "Could we still determine whether or not the earth is spinning on its axis?" The physicist promptly answers, "Yes, of course, the centrifugal and Coriolis effects can be meas-

[13] See F. Hoyle, Monthly Notices Roy. Astron. Soc. **108**, 372 (1948), and H. Bondi and T. Gold, *ibid.* **108**, 252 (1948); also, Hoyle, *The Nature of the Universe* (Harper and Brothers, New York, 1951).

ured on the earth's surface, and will indicate the earth's rotation." It would appear, then, that rotational motion has a certain absolute quality; we can say for certain whether a body is rotating or not. This situation is in contrast to that of linear motion. Whether a body is in linear uniform motion or not depends altogether on the observer's coordinate system, and there are no signs in space that will indicate whether a given coordinate system is moving or is at rest. This latter fact was emphasized by the special theory of relativity. It might be expected that circular motion would similarly be "relative" to the observer.

Mach's suggestion was that the phenomena associated with circular motion do in fact obtain only because the motion of the body in question is with respect to some other bodies. In terms of relativity theory, this would mean that the structure of space is determined by matter in the space, such that the Coriolis and centrifugal forces—the so-called circular forces—appear when a body is rotated in the space. The stellar and nebular masses thus determine the structure of space in such a way that the rotation of the earth gives rise to the circular forces; or, put in another way, the circular forces result from an interaction between the rotating earth and the fixed stars. One could equally well speak of the stars as rotating and the earth as being stationary; complete relativity of circular motion, analogous to the relativity of uniform linear motion, is thereby achieved. Likewise, ordinary linear acceleration would presumably require local forces because of the structure of space given by the masses in the space.

The situation is, however, that the complete "relativizing" of motion, in the sense just indicated, is not a part of the present theory of general realtivity. The most ambitious attempt to formulate a completely relativistic theory of rotational motion seems to have been made by H. Thirring.[14] He considered the effect of a rotating hollow sphere, and found that the general relativity equations indicated that there would be Coriolis and centrifugal forces at the center of the sphere, resulting from its rotation. The forces indicated were not large enough, however, by any means, to account for the circular forces found on the earth, for example, as a result of its rotation, even with liberal estimates for the mass density of the stellar matter that would presumably "fix" our spatial structure. Essentially, orthogonal space must still be assumed as a substratum in order to account for the observed forces. Thus, in the theory of relativity it is taken that space becomes Euclidean in structure in the limiting case of a complete absence of matter. It has been pointed out[15] that on a strict adherence to Mach's principle, relativity theory should indicate space to have no structure at all (or to have "disappeared") in the case of the complete nonexistence of matter. Einstein, however, has made the comment that "···the theory of relativity makes it appear probable that Mach was on the right road in his thought that inertia depends upon a mutual action of matter."[16]

It seems, then, that as far as our present knowledge goes, we should ascribe to space a certain absolute structure: that of three-dimensional orthogonality. There are deviations from this structure in the neighborhood of masses, to wit, the space-structure deviations associated with gravitational fields, and there are the very large-scale deviations which are associated with the cosmological structure of the universe. Both of these deviations are related to the presence of matter, in the manner described by the general relativity equations. But the basic three-dimensional, orthogonal character of space on which the deviations are apparently superimposed has not been shown to follow from any particular matter distribution, and must be taken as an independent, prior fact. This situation is hardly satisfactory, for the undeniable influence of space structure on the behavior of matter gives the impression that some elucidation of three-dimensional orthogonality, in terms of matter distribution, might be expected.

An approach to the relation of large and small that is quite different from the one just discussed should be mentioned. A number of physicists have been impressed by the approximate equality of various pairs of dimensionless ratios that may

[14] H. Thirring, Physik. Z. 19, 156 (1918); 22, 29 (1921).

[15] References are given in the note by A. J. Coleman, Math. Rev. 11, 467 (1950).

[16] Albert Einstein, *Meaning of Relativity* (Princeton University Press, Princeton, New Jersey, 1945), second edition, p. 100.

be constructed from physical quantities. Most noteworthy of these equalities is perhaps that between the ratio of cosmic to nuclear radii and the ratio of electrical and gravitational forces. Calculations on the basis of those cosmological models which indicate a determinate value for the radius of the universe have led to a value of the order of 10^{27} cm for the radius; the range of nuclear forces indicates a value of the order of 10^{-13} cm for the radius of the nucleus. The ratio of these radii is, then, $\sim 10^{40}$. Likewise, the ratio of the electrostatic force between a proton and an electron to the gravitational force between these two fundamental particles is of the order of 10^{40}. P. Jordan[17] sees this equivalence as expressive of some functional dependence between large- and small-scale aspects. Thus, he suggests that as the radius of the universe increases, as it seems to be doing, the constant of gravitation will decrease in value, so as to maintain the equality of ratios just noted.

The most extensive work with groups of dimensionless ratios of physical quantities has been done by Eddington.[18] His development seems most certainly to presume a conviction that the over-all features of the universe are intimately related to the small-scale features that we observe in the laboratory. Sometimes, to be sure, Eddington seemed to say that this relatedness has its origin more in the unity of method in the entity (i.e., man) who is describing both the large and the small of the universe than in an objective, existent unity. In any case, he has constructed a theory that relates the assumed size of the universe (taken to be closed and finite) and the total number of particles in it (calculated by purely *a priori* theory) to such atomic quantities as the fundamental electronic charge, electron and proton masses, and range of nuclear forces.

Neither Eddington's work nor that of others who have followed the possible clue of numerical ratios has as yet been sufficiently successful or sufficiently accepted generally, to become a part of present day physical theory. One feels, though, that these interesting speculations, including Mach's suggestion too, represent a groping after some great truth about our physical world. We can hope that the undergraduate physics student of the year 2000 will see the facts which have occasioned these contemporary speculations as straightforward consequences of that truth, and will wonder why we of 1951 were so stupid as not to put together the obvious.

[17] P. Jordan, Nature **164**, 637 (1949). We are here presenting only a single facet of Jordan's speculations.

[18] An account is given by E. T. Whittaker, in his *From Euclid to Eddington* (Cambridge University Press, London, 1949), Part V.

TO THE END OF SPACE

In conclusion, we point out an alteration in our concept of the continuity of space that is today being considered as a result of developments in quantum and nuclear physics. Both Newtonian and relativistic mechanics, as well as the classical electromagnetic theory of Maxwell and Faraday, assume that space is continuous, in the mathematical sense of that word. This assumption means that properties of space are defined at every point of the space; thus, Maxwell's electromagnetic equations and Einstein's equations for the relation of mass to space-time structure are field equations which ascribe certain properties to any point of the space over which they are defined. There is the obvious difficulty, however, that these equations are not defined at the points of origin of the electromagnetic and gravitational fields, where the distance from the origin points is zero. Hence, those who would calculate interactions at the level of elementary particles are faced with the considerable obstacle of divergences and infinities resulting essentially from the singularity points in the customary fields at their points of origin. This same difficulty occurs in the consideration of the nuclear forces between particles in terms of so-called meson fields.

The suggestion has been made by W. Heisenberg and others that there is in nature a "fundamental length," l, of the order of 10^{-13} cm, such that space in intervals of this magnitude or less ceases to have the properties generally ascribed to it. Thus, to quote Heisenberg,[19] "If the future theory contains such a constant (as l) in whatever form, it is natural to assume that the usual correspondence between the classical wave description and its quantum theoretical analogue

[19] Heisenberg, "The present situation in the theory of elementary particles," in his *Two Lectures* (Cambridge University Press, London, 1949), p. 12.

only holds for distances very much greater than l, but fails in the region of smaller distances." The elementary length would presumably give a theory in which there would be a natural "cut-off" of fields at their origin points. We would then find that space comes to an end in the very small: that if we go to a small enough region, we no longer find space to be the physical entity that our physics and geometry now describe for larger regions.

The "fundamental length" is only a possibility in today's physics; there seem to have been recent successes in the quantum theory of electromagnetic fields and their interactions with charged particles, and perhaps a complete theory of elementary particles and their interactions will yet be achieved without requiring a fundamental change in small-scale space structure. There is, however, a suggestion of the need for such a change even in the uncertainty relations of quantum theory. That a particle's position and velocity, for example, cannot be specified at a point is indicative of a concept of space, as well as of a wave-particle entity, that is incompatible with classical concepts. A relevant remark that gives insight into the problem is one by A. N. Whitehead: "Continuity concerns what is potential; whereas actuality is incurably atomic."[20]

EPITOME

We can summarize our account of space in today's physics by comparing our present concepts with those which dominated physics as long as Newtonian mechanics was unquestioned. The space of Newtonian physics is a space of three orthogonal dimensions that is everywhere exactly described by Euclidean geometry, and it extends without end in all directions; bodies in the space are "rigid" and unaffected by any spatial motions; bodies, and forces like gravitation, are contained in the space, but space contains or transmits all physical things without in any manner being changed by those things; space is, indeed, metaphysical rather than physical, a subject for the philosopher's cogitation on the common aspects of all existence rather than for the physicists's investigation of process and structure in natural phenomena. In the physics of 1951, the orthogonal, three-dimensional character of space has been modified to allow both some local curvature in the neighborhood of masses and possible over-all cosmological curvature; spatial dimensions of bodies and the time rates of physical processes are changed by motion relative to the observer; space structure is changed by the presence of matter, and space structure gives rise to accelerating, gravitational forces; the physicist's detailed, mathematical description of natural phenomena requires explicit knowledge and decision by him about the nature of space.

The change in the conception of physical space which has occurred in this century may be described in a word as a change from an emphasis on an "absolute space" to a consideration of a "relational space." These two contrary views of space have been discussed since the days of the Greek philosophers.[21] The "absolute" doctrine—notably presented in physics by Newton—holds space to be an entity above and independent of the physical things which are in space. According to the "relational" view, the properties of space are essentially defined in terms of objects and relations between objects; space would not exist apart from the objects in it, just as objects would not exist apart from the space that contains them. The relational doctrine was held by Aristotle in Greek times, and was argued against Newton in his own day by Leibnitz. In modern times Riemann and Einstein have brought physics toward such a view. It is, of course, Einstein who is chiefly responsible for our new ideas of space. These ideas have been established, not by Einstein's defending a philosophical position, but by his successfully using the ideas in struggles with difficult problems of physics. Newton, of course, did likewise with absolute space, some two and a half centuries earlier, in the less inclusive physics of his day.

[20] A. N. Whitehead, *Process and Reality* (Macmillan Company, New York, 1929), p. 95.

[21] A discussion of the two conceptions will be found in almost any book treating of philosophical questions in physics. A discussion in the light of contemporary physics is given in Henry Margenau's *The Nature of Physical Reality* (McGraw-Hill Book Company, Inc., New York, 1950), Chapter 7. An historical treatment is given by Sir Edmund Whittaker, reference 18, Secs. 2–7, 42.

NOTES AND DISCUSSION

Mathematics in the Undergraduate Physics Curriculum

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THE results of a survey on mathematical emphasis in undergraduate physics quoted in a recent article by Phelps[1] show a considerable awareness on the part of physics curriculum planners of the need for mathematics in physics courses. It seems that this problem has perhaps not received the discussion in this country recently that its importance suggests. In England, for example, some publications on the subject[2,3] reveal considerable work and thought. Perhaps there has not been more discussion of the topic in the United States recently because of the wide variation in mathematical emphasis, as reported by Phelps, with some schools requiring a considerable amount of mathematical knowledge, and others emphasizing the experimental and descriptive approach to physics.

It does seem, though, that the subject matter of physics is becoming more extensive and more mathematical these days, and it would be reasonable to require more mathematics of the student than formerly. This will be especially true of those undergraduate students planning to do graduate work (currently about 40 percent, according to the results of Phelps' questionnaire), but will also be true for those planning to stop at the bachelor's degree.[4] In this country a distinction is seldom made in the curricula of the two, except perhaps in recommended electives.

To be specific, let us consider the mathematical requirements of two courses often given for junior and senior physics majors—sound and quantum mechanics. These one-semester courses are both taught at the University of San Francisco, a liberal arts institution giving only the bachelor's degree in physics at present. Sound is elective; quantum mechanics, required. Sound is given at about the level of difficulty of Kinsler and Frey's text.[5] To choose a textbook for the quantum mechanics course is something of a problem, but the level of the course is about that of the texts of Pauling and Wilson[6] or Rojansky.[7] Sound has only the two-year lower division sequence in beginning physics plus calculus as prerequisites. Quantum mechanics has these plus introductory atomic physics (text: Semat[8]) and the first semester of intermediate mechanics (text: Synge and Griffith[9] or Lindsay[10]). In addition, the students are required in their junior year to take a year of differential equations, so they will have had this course or will be taking it concurrently. In none of these prerequisite or concurrent courses, however, will the students encounter such topics as partial differential equations solved by separation of variables, for example, the wave equation and Schrödinger's equations; Fourier series expansions (and even Fourier integrals in quantum mechanics); improper integrals; elementary complex variable theory; and orthogonal sets of functions in general and expansions of other functions in terms of them. All of these topics appear in sound or quantum mechanics. The student need not have had a complete, rigorous, mathematical treatment of them, however, in order to be able to use them in the courses.

As a result of the students' lack the physics instructor must spend considerable time helping them with the necessary mathematical concepts, which seriously cuts down the amount of physics that can be covered; or else the instructor must leave much of the mathematics for the students to pick up themselves, which they may or may or may not be able to do. The mathematics often makes relatively unintelligible for the students a course otherwise full of physical content.

Physics majors sometimes find the content of mathematics courses given in mathematics departments too rigorous and limited in scope for their purposes. Obviously if all the mentioned topics were treated in rigorous detail, the students could not possibly finish them in time for the sound and quantum mechanics courses.

Recently a somewhat experimental course in mathematical physics has been instituted in the physics department at the University of San Francisco. It is being given as an elective, two-part course with each part independent, so that a student can take either part or both while a junior and senior. One difficulty is that the physics curriculum is already crowded, so that it is hard to add required or even optional courses. Those students planning to do graduate work are especially urged to take at least one part. Such topics as are listed in the preceding paragraph are treated in the mathematical physics course. A completely satisfactory text is not presently available. Margenau and Murphy's book[11] comes close: for some topics the book is near the desired level of difficulty of the course; for others, it is somewhat more difficult than the course. Vector and tensor analysis is given as a separate year course in our mathematics department and is an elective for physics majors. The mathematical physics course is not merely a survey course in mathematics, since the topics are treated as they appear in the framework of physics. The students are shown in what parts of physics the mathematics arises, and then how to handle it. The students who have taken one or both parts of the course report that it has helped them to understand the mathematics and physics of other courses. At worst the students are at least "exposed" to the methods of mathematical physics.

Another change in the curriculum of physics majors at the University of San Francisco will soon be made: the mathematics department is planning to revise the upper division differential equations course. Additional topics of interest to physics majors will be included.

The difficulty students have with the mathematics of upper division physics courses is merely one part of the problem of how much mathematics to give in an undergraduate physics curriculum. Our freshmen now start out in their first semester (as they do in many other schools) taking calculus, so that a limited use of it can be made

concurrently in lower division physics. One must strike a balance, of course, between not enough mathematics in the physics curriculum and too much. In the latter case the student may rely on mathematical steps entirely, which sometimes inhibits a complete understanding of the physical principles involved. But it seems at present that in view of the stiffening of the mathematics required to understand physics, too little rather than too much mathematics has been taught.

[1] J. Phelps, Am. J. Phys. **19**, 122 (1951).
[2] "The Teaching of Mathematics to Physicists," Report of the British Mathematical Association and the British Institute of Physics (The University, Reading, England, 1944).
[3] L. Rosenhead and C. Strachan, Am. Math. Monthly **55**, 403 (1948).
[4] Some years ago the Committee on the Training of Physicists for Industry of the AAPT reported on the need for satisfactory mathematical training of physicists in industry. See Am. J. Phys. **8**, 124 (1940). See also on the same subject R. G. D. Richardson, Am. J. Phys. **11**, 67 (1943).
[5] L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics* (John Wiley and Sons, Inc., New York, 1950).
[6] L. Pauling and E. B. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1935).
[7] V. Rojansky, *Introductory Quantum Mechanics* (Prentice-Hall, Inc., New York, 1938).
[8] H. Semat, *Introduction to Atomic Physics* (Rinehart and Company, Inc., New York, 1946).
[9] J. L. Synge and B. A. Griffith, *Principles of Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), second edition.
[10] R. B. Lindsay, *Physical Mechanics* (D. Van Nostrand Company, Inc., New York, 1950), second edition.
[11] H. Margenau and G. M. Murphy, *The Mathematics of Physics and Chemistry* (D. Van Nostrand Company, Inc., New York, 1943).

Note on the Quantum Theory of the Doppler Effect for a Moving Observer

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IN most textbooks the formula for the special relativity Doppler effect is derived from the relativity time dilatation equation. Though it does not show anything new it is perhaps worth while to prove the same equations with the help of the laws of conservation of momentum and energy alone, which laws are more familiar to many students than the relativity time dilatation equation.

This derivation starts from the corpuscular theory of light and considers the Doppler effect as a collision process in a way similar to the corpuscular derivation of the formula for the Compton effect; in this collision process momentum and energy are conserved. Some textbooks give this derivation for a moving source consisting of a moving excited atom (e.g., a canal-ray particle) but only for the classical first-order Doppler effect.[1] A consequent application of the relativistic laws of conservation of momentum and energy gives the correct relativistic formula[2,3] for this case.

At first sight the case of a moving observer seems to be more difficult, as one has to find a quantum description of a moving observer. It will be shown below that the corresponding formula for the case of a moving *observer* can be derived by replacing the observer by a moving absorbing atom. The observed frequency then follows from the amount of energy absorbed by the atom.

In order to calculate this energy one has to take into account that a quantum hf (h is Planck's constant, f is the frequency) moving with the velocity of light c has a momentum hf/c and that an amount of energy $E_0=hf_0$, available in the atom after the absorption process, increases the rest-mass of the atom by an amount hf_0/c^2.

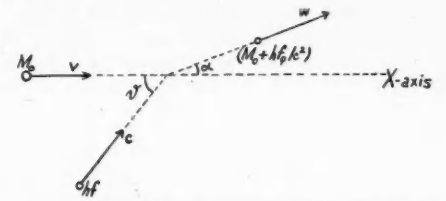

FIG. 1. A moving atom M_0 absorbs a quantum hf. During the collision, the directions of motion change.

Let M_0 be the rest mass of the atom (Fig. 1) before absorption has taken place, $E=hf$ the energy of the quantum before absorption, and $E_0=hf_0$ the energy actually absorbed by the atom. Let the quantum move with the velocity c in a direction making an angle ϑ with the positive x axis. Let the atom move with a velocity v in the direction of the positive x axis before the absorption of the quantum hf; because of the recoil in the absorption process the velocity of the atom will be changed to a value w and the direction of its motion will now make an angle α with the positive x-axis (all velocities have to be taken relative to the source, of course). We then have for the law of conservation of the x and y components of the momentum ($\beta=v/c$; $\gamma=w/c$);

$$M_0\beta c(1-\beta^2)^{-\frac{1}{2}}+(hf/c)\cos\vartheta = (M_0+hf_0/c^2)\gamma c(1-\gamma^2)^{-\frac{1}{2}}\cos\alpha, \quad (1a)$$

$$(hf/c)\sin\vartheta = (M_0+hf_0/c^2)\gamma c(1-\gamma^2)^{-\frac{1}{2}}\sin\alpha, \quad (1b)$$

whereas according to the law of conservation of energy:

$$M_0c^2(1-\beta^2)^{-\frac{1}{2}}+hf=(M_0+hf_0/c^2)c^2(1-\gamma^2)^{-\frac{1}{2}}. \quad (2)$$

Dividing Eqs. (1a) and (1b) by c and Eq. (2) by c^2, and subtracting the sum of the squares of Eqs. (1a) and (1b) from the square of Eq. (2) one obtains

$$M_0^2+2M_0(1-\beta^2)^{-\frac{1}{2}}(hf/c^2)(1-\beta\cos\vartheta)=(M_0+hf_0/c^2)^2, \quad (3)$$

or

$$f(1-\beta\cos\vartheta)(1-\beta^2)^{-\frac{1}{2}}=f_0[1+\tfrac{1}{2}hf_0/(M_0c^2)]. \quad (4)$$

If the atom is initially at rest ($\beta=0$) we find

$$f=f_0[1+\tfrac{1}{2}hf_0/(M_0c^2)]. \quad (5)$$

The small difference between f_0 and f (which for most purposes may be disregarded) is the result of the recoil of the atom, initially at rest, in the absorption process. Disregarding this difference we have, for the "observed" frequency f_0,

$$f_0=f(1-\beta\cos\vartheta)(1-\beta^2)^{-\frac{1}{2}}, \quad (6)$$

which is the correct relativistic formula.

Michels[4] treats this problem from a slightly different angle. He introduces two observers O and O' moving with a speed u relative to each other. Consequently he not only has to make use of the laws of conservation of momentum and energy, but he also has to apply the relativistic trans-

formation equations relating the relative velocities of a moving particle with respect to the observers O and O'. The advantage of this method is the same as for the ordinary relativistic treatment, *viz.*, that he obtains formulas of a more general nature which may be applied to other phenomena besides Doppler effect.

In the foregoing discussion, however, only one observer is introduced, and for that reason the laws of conservation of momentum and energy *alone* are sufficient to solve the problem of the Doppler effect.

[1] E. Grimsehl, *A Textbook of Physics* (Blackie and Son Limited, London, 1945), Vol. 5, p. 240.
[2] E. Schrodinger, Physik. Z. **23**, 301 (1922).
[3] A. vanderZiel and W. deGroot, Physica, (s'Grav.) **12**, 499 (1946).
[4] W. C. Michels, Am. J. Phys. **15**, 449–450 (1947).

Motor-Driven Vibrator Units for the Measurement of Capacitance

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THE vibrator units described in this paper are adaptations of the Amglo constant speed, battery-driven motor.[1] The first one operates from a 6-volt battery while the second adaptation operates on a 110-volt, 60-cycle ac circuit leaving the reed contactor, ordinarily employed as the control for the motor, free to charge and discharge a condenser.

The essential features of the Amglo motor, used in the construction of the vibrators, are shown in Fig. 1, which is a reproduction of a photograph of a motor-driven vibrator operating on an ac circuit. The electrical circuit

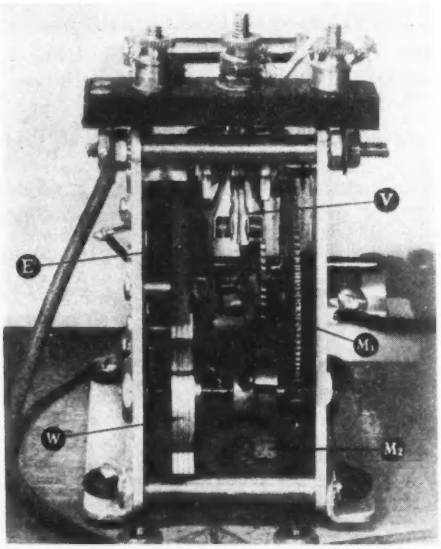

FIG. 1. Vibrator unit for operation on an ac circuit.

of the Amglo motor consists of a stationary electromagnet E in series with a 6-volt battery and a vibrator make and break unit V. The reed of the vibrator carries a small Alnico magnet M_1 and is actuated by an eight-pole plate magnet M_2. The shaft, on which M_2 is mounted, carries a soft iron wheel W, with four projections, which rotates in front of the poles of the stationary electromagnet E. The intermittent current in the electromagnet acting on the wheel W actuates the motor. A train of gears is provided with the motor reducing the speed in two steps of ten.

For the first adaptation the gear train was removed and in its place an extra vibrator unit was mounted. Electric connections from the reed and each of the two contacts were made to three insulated binding posts. The motor operated satisfactorily on a 6-volt battery and the second vibrator, which was electrically independent of the motor, gave a satisfactory charge and discharge device for experimental work with condensers. By means of adjustments provided with the vibrator the cyclic frequency could be adjusted to equal 60 cycles sec^{-1}.

The second of the units, illustrated in Fig. 1, was developed for operation on a 60-cycle ac line. The electrical circuit employed for this work is shown in Fig. 2. The

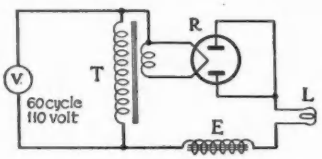

FIG. 2. Electrical circuit for the ac-operated vibrator unit.

electromagnet E of the motor was disconnected from the vibrator and connected into the plate-to-filament circuit of a R.C.A. No. 80 radiotron tube. A lamp L was connected into the circuit to control the electromagnet current. With the half-wave rectified supply in the electromagnet the unit operates as a synchronous motor when it has been started. To carry out this operation, a torque is applied simultaneously to the gear wheel and magnet wheel and the speed increased until the electrical circuit takes control. Satisfactory operation was secured with an equivalent current of 0.11 amp. Adjusting screws are provided on each side of the vibrator, and these were set so that the time of contact, on the make and break, respectively, was approximately one-third of the cyclic time. This time was measured by connecting a resistance meter between the center and each side in turn.

The unit was tested by employing it in the Fleming and Clinton direct deflection method for the measurement of a 0.001-μf standard mica condenser. The 22½-volt battery employed was connected through a reversing switch, and deflections were noted on both sides of the zero with the average of the two considered as the correct reading. The deflection secured with the insulated terminal of the condenser disconnected was employed to correct for the capacitance of the circuit. The corresponding equation connecting the average net galvanometer deflection θ_1, the capacitance C, the cyclic frequency N, the galvanometer constant k and the battery emf E is

$$k\theta_1 = NEC. \qquad (1)$$

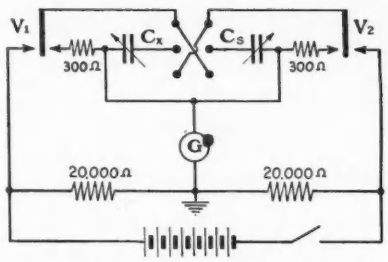

FIG. 3. Bridge circuit for the precision calibration of a variable air condenser.

To eliminate k and E the condenser and vibrator were replaced by a resistance R, and the average galvanometer deflection θ_2 was noted. The corresponding equation is

$$k\theta_2 = E/R. \tag{2}$$

Combining Eqs. (1) and (2) gives

$$C = \theta_1/\theta_2 NR. \tag{3}$$

As a check on the accuracy of this method, it was employed to measure the capacitance of a 0.001-μf standard condenser in terms of a resistance R consisting of ten 1-megohm precision resistors in series. Using a Weston table galvanometer with a sensitivity of 0.1 μa per division as a detector, four determinations gave an average value of 0.000998 μf for the capacitance of the standard with a maximum deviation of 0.6 percent. In many respects this method of measuring capacitance in terms of resistance is much simpler than Maxwell's bridge method, usually recommended for the purpose.

For the comparison of an unknown air condenser with a variable standard a modification of Kelvin's method of mixtures was devised. The circuit, illustrated in Fig. 3, employs two vibrator units V_1 and V_2 in place of the combination of switches or special commutators[2] which were used for the method of mixtures. This circuit was employed with a G.R. type 539B variable air condenser as C_x. For various dial settings of C_x in the range of 100 to 1025 $\mu\mu$f, its capacitance was determined by comparing it with a precision standard variable air condenser C_s for which a chart was available. The resistances in the ratio arms were kept equal, and by means of the reversing switch the circuit positions of the vibrators were interchanged. Each measurement for the condenser C_x was considered as equal to the average of the readings for C_s required to balance the circuit. Accurate determinations were secured using a galvanometer with a sensitivity of 0.038 μa/mm and a resistance of 320 ohms. A 135-volt B battery was used with the bridge.

As a check on the accuracy of the work the values measured on the bridge were compared with a ten-point calibration covering the range from 109 to 1026 $\mu\mu$f. In two tests, the maximum and average deviations between the measured and calibrated values were, respectively, 1.0 and 0.03 $\mu\mu$f. The main advantages of this method are that elaborate shielding is unnecessary and only two precision resistors are required.

Measurements of the capacitance of four vibrators used in the laboratory gave values varying from 50.0 to 51.1 $\mu\mu$f. The insulation resistance between terminals as checked on a megger was much greater than the 100-megohm maximum reading on the scale of the megger.

No attempt has been made to consider other uses of these vibrators since many other types are available especially for radio circuits. The work outlined in the paper represents special uses for which the vibrators are eminently suited.

[1] Amglo Corporation, 4234 Lincoln Avenue, Chicago 18, Illinois.
[2] H. L. Curtis, Bull. Bur. Standards **6**, 431 (1910); Ainslie, Am. Phys. Teacher **6**, 325–326 (1938).

Experimental Test of Fresnel Diffraction Theory with Photomultiplier Tube*

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THE relative intensities of a Fresnel diffraction pattern, such as the shadow of a straight edge, can be predicted by Cornu's spiral. The prediction can now be tested by measuring the intensities with a photomultiplier tube. Details are given below of an experiment which works well and gives measurements agreeing with theory within 2 percent or better.[1]

Green light from a mercury arc is focused on a source slit, proceeds to an obstruction such as a straight edge, and continues to an observing region where it enters a movable photomultiplier tube (see Fig. 1). The current from the tube is measured with a mirror-and-scale galvanometer, whose reading is plotted against the lateral position of the tube.

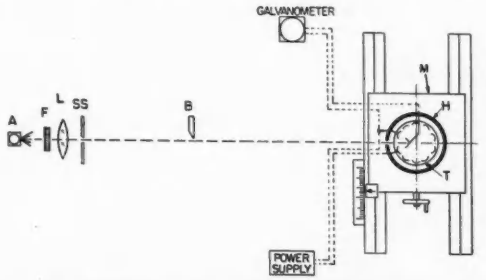

FIG. 1. Diagram of apparatus in which the RCA 931-A photomultiplier tube T is used to explore the distribution of intensity in the diffraction pattern of a straight edge B. The source A is a high-pressure mercury arc lamp; F is a filter, L a lens to focus the light on the slit SS. The distance from SS to T is about 150 cm. The tube T can be moved on accurate ways.

The green light of wavelength 5461 angstroms is obtained from a high pressure mercury lamp A whose radiation is passed through a filter F and focused on a vertical source slit SS about 0.05-mm wide.

A razor blade B serves as a vertical straight edge and is located about midway between SS and T, the distance between these two being about 150 cm.

The RCA 931-A photomultiplier tube T is mounted on, and moved by, a micrometer screw M which measures

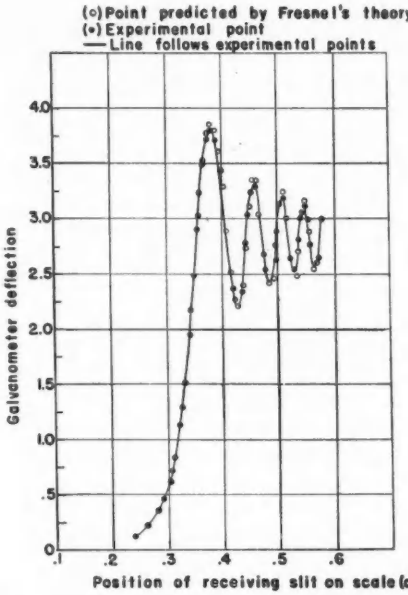

FIG. 2. Experimental determination of the variation in intensity in the diffraction pattern of a straight edge. Open circles are points predicted by Fresnel's theory; full circles are experimental points through which a continuous smooth curve has been drawn.

the lateral displacement of the tube across the diffraction pattern. The tube is completely covered by a black paper hood H except for a vertical slit about 0.1 mm wide cut in its face with a ruler and razor. Resistors of 1 megohm each are put between the dynodes of the tube, and it is fed by a 60-cycle power supply P which provides 800 V constant to within 1 percent. The current from the tube is measured on the galvanometer G whose sensitivity is about 0.2 microamp per centimeter at 1 meter.

The power source and arc are turned on and allowed to settle down for a half hour or more. The hood slit and source slit are made approximately vertical by eye. The razor blade is rotated until it also is vertical by tapping it gently until the galvanometer reading for the first maximum is itself at a maximum. The blade is then clamped firmly in that position. The clamp and blade are removed from the optical bench for a blank measurement which consists of several readings across the unobstructed beam. Next, the blade and clamp are put back and the intensity measured for various positions of the tube across the resulting diffraction pattern. Finally, the blank measurement is taken again.

The galvanometer currents are plotted against the lateral displacement of the tube across the diffraction pattern. It is difficult to measure the position of the geometrical shadow accurately. This may be avoided if we assume that it occurs at the place where the intensity is one fourth of the blank reading. Then a set of relative intensities predicted by Cornu's spiral for the distances and wavelength used are calculated and multiplied by a scale factor to make their unobstructed intensity agree with the experimental blank reading. These theoretical intensities are plotted on the experimental graph. For a specimen see Fig. 2.[2] It is found that the graph of the experimental points agrees well with the theoretical points, both in the lateral spacing of maxima and minima and in the heights of the maxima and minima.

[*] Condensed from a senior thesis in partial fulfillment of the requirements for the A.B. degree, Princeton University.

[1] Tests have been made by: Lyman, using a photographic method, Proc. Nat. Acad. Sci. **16**, 71 (1930); and by Hause, Woodward, and McClellan, using a vacuum photocell and electrometer tube, J. Opt. Soc. Am. **29**, 147 (1939).

[2] Both the source slit and the hood slit have finite widths. The experimental method thus integrates superimposed patterns and smooths out the maxima and minima slightly.—Ed.

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LETTERS TO THE EDITOR

Short Laboratory Manuals Are Efficient

MANY schools use a blank form for a report of a laboratory experiment in the sciences. The student performs the experiment and fills out certain spaces and columns in a tabular sheet. He may then be questioned orally by the teacher and the report approved. Some schools have had this plan since the turn of the century.

But far too many colleges and secondary schools stick to a long-winded discussion and description of the experiment which in many cases only confuses the student when he reads it before the laboratory period. Five minutes with the teacher in front of the apparatus is worth ten pages of descriptive material. And no matter how much descriptive material is supplied, the teacher at the beginning of the period must always clarify, motivate, and start the experiment with the class. The student spends two hours or so doing the experiment and then another two to six hours writing up a very long-winded discussion, paraphrasing the printed pages of information, writing the answers to set questions, tabulating data, drawing graphs, etc. Or rather we should say that he is supposed to write up the experiment. Many schools use the same laboratory manual year after year, and every fraternity and sorority keeps files of back experiments for the use of their novitiates. Then too, students occasionally sell the output of their laboratory work or pass it on to a friend. So in a good many cases the experiments are copied mechanically from year to year. The student goes through the motions in the laboratory not caring whether he does a profitable experiment or not. He knows that he has an approved experiment that he can fall back on and reproduce as his own.

In contrast we have the one-page fill-in sheet that the student completes in the laboratory in full view of the instructor. The instructor can see if he gets any help from anyone, how he tackles the job, how long it takes him, and can interrogate him orally at any time to see if he really understands what he is doing. These one-page fill-in sheets can easily be varied from year to year while the job of rewriting long laboratory experiments in a manual is a major project.

There is no necessity for including at this point a sample of a long-winded drawn-out discussion-type experiment as most of us are only too painfully aware of this abomination. The author, who teaches in one of the largest colleges in the largest city in the country, can cheerfully supply laboratory manuals from his institution in which every experiment in elementary physics is seven to ten pages long. The students never visualize the experiment in its entirety due to the excess verbiage,—the old story of the forest and the trees.

As a contrast the author suggests a typical one-page fill-in type report for that ever popular experiment, the Atwood Machine, a report that the student completes during the laboratory period.

The One-Page Tabular-Type Experiment to be Filled Out in the Laboratory—the Atwood Machine

1. Stand at your place and identify the different parts of the apparatus as the teacher discusses the experiment.
2. Decide which is going to be the heavier side and put enough weight on that side so that it will descend freely. Record the height, and time the descent twice. Note the mass difference.
3. Increase the weight-difference by small amounts twice more, and make two time trials for each case. Record times and mass differences.
4. Make a graph with the mass difference as abscissa and the acceleration as ordinate. Read off and record the friction correction. The acceleration is obtained from the usual formula, $s=\frac{1}{2}at^2$.
5. Substitute in Newton's second law of motion and solve for g, the acceleration due to gravity.

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Entropy and the Uncertainty Principle in Social Physics

I AM emboldened by Professor Stewart's suggestive comment[1] not to be timid with one's ideas, even though he warns against the dangers of analogy. I wish in this note to propose that the principles of entropy and uncertainty may have some functional value to the sociologist or the social physicist, and that if he has not yet taken the language he might with profit do so, and indeed adapt the mathematical formulation to his own data.[2]

It appears, on the surface at least, that the principle of entropy may be utilized by the sociologist in his inquiries on *social disorder*. For example, consider a social organization possessing a hierarchy of classes or social statuses (such as the Indian caste system or the status hierarchy of the Navahos). Now once this organization, assumed in the first instance as possessing reasonable order quite arbitrarily, has been disturbed by, say, the death of the leader, the failure of the economic system, or by cultural conflicts, the original order and structure are quite impossible to restore. In brief, the entropy of the system has increased and the process is practically irreversible. It is quite impossible to restore the original configuration.

Concerning the uncertainty principle, its application appears even more likely. Consider, for example, an inquiry made by poll where the data are accumulated by asking questions of people concerning their opinions or their status. It is obviously clear that the answers are prejudiced and biased by the poll taker as a personality. When then do we get the individual's intrinsic opinion? To what extent are the replies uncertain? How can we minimize this uncertainty? Freud, of course, suggested hypnotism, whereby the subject would be divorced from his environment.

Consider an anthropological study of primitive societies. The investigator must be mindful always of the native's definition *of him* as a personality. This definition colors the answers to the questions by some uncertain but perhaps calculable amount. Anthropologists have recognized this aspect in their investigations of primitive peoples but have never assessed its weight mathematically. Even when scholars are "accepted" by tribes or isolated peoples and become, so to speak, one of them, the fact remains that personal and group history separate them and uncertainty in the findings prevails.

The mathematical formulation of both entropy and uncertainty may need some modification by the social physicist but the principles appear quite applicable.

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[1] John Q. Stewart, Am. J. Phys. **18**, 253 (1950).
[2] Other aspects of this question have been considered before. See, for example, H. C. Dickinson, Phys. Rev. **45**, 741(A) (1934).

Wavelengths of Damped Waves

IN discussions of wave forms and wave propagations it is of interest to contrast the "wavelength" of a spherical wave expanding outward from a point source S with the wavelength of a plane wave exponentially damped as a consequence of absorption and scattering. The equation $y=(A/r)\sin(kr-\varphi)$ expresses the wave form at a given instant in the former example, so far as the disturbance along a particular radius r is concerned. The symbol A represents the amplitude at unit distance from S, and k equals $2\pi/\lambda_0$, where λ_0 is the constant wavelength that would exist if the amplitude were constant. The linear inverse diminution of the displacement and amplitude is a consequence, of course, of the inverse square law associated with the intensity. Setting the derivative dy/dr equal to zero and solving the resulting implicit equation, $kr=\tan(kr-\varphi)$, we can easily show that the intervals between maximum displacements are not constant, are less than λ_0, but approach λ_0 asymptotically as r approaches infinity.

The equation $y=Ae^{-\alpha x}\sin(kx-\varphi)$, (or $y=Ae^{-\alpha x}\times e^{i(kx-\varphi)}$), represents the instantaneous form of an exponentially damped plane wave, where α is a coefficient that arises from absorption and/or scattering. Setting the derivative dy/dx equal to zero, we obtain the equation, $k=$constant. Therefore, in this instance the intervals between maxima not only are constant but, moreover, are equal to the wavelength λ_0 of a corresponding undamped wave. Therefore, we may speak, without qualification of *the wavelength* of an exponentially damped plane wave as well as of an undamped one.

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ANNOUNCEMENTS AND NEWS

Book Reviews

Semiconductors. D. A. WRIGHT. Pp. 130. John Wiley and Sons Inc., New York, 1950. Price $1.75.

In the last 30 years the interest in and use of semiconductors have exceeded those in all previous time. While the properties of semiconductors are by no means completely understood, enough has been learned and the theory of their behavior has been sufficiently advanced to call for comprehensive expositions at various levels. This monograph is a brief and concise textbook intended for students of the electron physics of solids. It concerns itself especially with the theory of electron and hole flow in semiconductors, and across the boundary between them and metals or a vacuum. The presentation is well organized and aims to explain the great variety of phenomena quantitatively in terms of formulas based on band theory. The advanced student will find numerous occasions to differ with the author's interpretation. Although the beginner may be misled in a few instances, he will be compensated by the direct and explicit treatment. The serious student will find that semiconductors are quite complex and that considerable study will be required to apply the theory with confidence to specific instances. However, the text will furnish him with sufficient leads and simulation to start him off on his explorations.

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Thermodynamics of Fluid Flow. NEWMAN A. HALL. Pp. 278+x. Prentice-Hall, Inc., New York, 1951. Price $5.50.

In reviewing this book for the physicist reader, the reviewer feels that the first thing he ought to be told is that this book is written for mechanical engineers by a mechanical engineer. The physicist reader will find disconcerting points regarding some of the conventions adopted (e.g., discussion of units and dimensions on page 3), some of the mathematical notations used (e.g., pages 38 and 39), and some inadequacy in the discussion of detailed mechanism (e.g., the discussion of turbulent flow). However, he will find that, despite these points, the information given in this book will be useful to him in solving practical problems of nearly one-dimensional flow through a pipe of nonuniform cross section. Such problems come up very often in practice, but they are so complicated that any detailed analysis of the mechanism would certainly be beyond the scope of the present book. But all phases of such problems are treated in this book by engineering methods. For example, Chapter 2 deals with wall friction, Chapter 8 deals with the variation of the cross section of the pipe, Chapter 9 deals with the formation of shocks and Chapter 11 deals with heat exchange. The reader will probably find in Chapter 10 (dealing with flow measurement and control) and in Chapter 12 (dealing with propulsion system) an introduction to the rather practical phase of flow problems.

The methods of analysis are based on the principles of mass, energy, and momentum. These and some other considerations of the basic properties of the fluid are presented in Chapters 2–7.

The discussions in this book are carried out in great detail. Anybody with a sound two-year course in college physics could probably read the book with little difficulty. This fullness of presentation, together with the large number of problems collected at the end of each chapter, should give the reader a feeling of concreteness and a readiness to attack practical problems.

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A Hundred Years of Physics. WILLIAM WILSON. Pp. 519. Gerald Duckworth and Company, Ltd., London, 1950. Price $3.50.

In reviewing a book of this kind, one should first of all consider the audience for whom it is written. This is one of the new Hundred Years Series covering a wide field of interests including a variety of sciences, humanities, and fine arts. Presumably any book in this series is meant to give an educated man a survey of the field covered in the last hundred years, it being tacitly assumed that the reader has little familiarity with it, and indeed may have less knowledge of it than one is expected to retain after an introductory university course in that subject. If this is a valid assumption, then this book must be regarded as being pitched at too high a level. The reviewer has often thought that a book in science should be appraised in the light of the answer to this question: Can I understand the presentation given without repeatedly drawing on my prior knowledge of the field? If this question be asked in regard to the

book being reviewed, it must be admitted that it hardly fulfills its purpose of giving the general educated reader a real understanding of the growth of physics during the century now ending.

In the preface the author states that "to understand most of this book the reader needs very little knowledge of the science of physics and a very slight mathematical equipment." This is surely too optimistic. Among the topics discussed are van der Waals' equation, Boltzmann's equation, group and wave velocities, the Lorentz transformation, Planck's blackbody radiation formula, and the stationary principles (least action, Hamilton's and Fermat's). The reviewer believes that even the better than average undergraduate student who has had two or three stiff courses in physics would not find the book easy going. Indeed, it might be used with profit by a graduate student preparing for his comprehensive examination in physics as a summary and index of topics.

However, there can be no criticism of the subject matter itself which is well selected and broad in scope. Dr. Wilson has been very successful in treating with adequate emphasis the many fields that add up to the physics of the century now ending. Our present-day enthusiasm for and concentration on quantum mechanics, nuclear physics, and the solid state tends to make us overlook many fields which in their own decades intrigued the best minds in physics. Dr. Wilson's presentation is very welcome in that it gives us an excellent perspective of physics as a whole.

Chapters 1 and 2 deal with thermal motion, the equivalence of heat and work, temperature scales, thermodynamics, statistical mechanics, and related topics. Today most of these are regarded as obvious and well understood; yet, as these chapters clearly show, there was a time when these ideas presented great difficulties, and advances were made hesitatingly and only as a result of sustained effort.

Chapters 6 and 7 cover in a very complete fashion a wide selection of topics in the field of light. Certain subjects such as the convection of light (Fresnel) which nowadays receive little attention are treated in some detail.

Chapter 8 deals with the rather dreary matter of units and absolute measurements which, while of importance to the professional physicist, are not likely to evoke much interest in the general reader.

In Chapter 9, Maxwell's electromagnetic theory is developed carefully. It should repay the general reader who is willing to give the necessary time to the exposition as it is presented.

In Chapter 10 we have a brief but clear account of certain experiments which, at the time they were made, appeared to demand the existence of an aether to interpret them. While physicists have long ceased to concern themselves with the aether, a book on the history of physics cannot ignore a topic which engaged the earnest attention of the leading physicists for several decades in the last century. Chapter 11 on relativity naturally follows. It is well done and should be clear to the reader who has the necessary mathematical preparation.

After Chapter 12, which is a brief outline of Einstein's general theory of relativity, we come to Chapter 13 where there is a good straightforward account of the main features of the phenomena of radiation and the need for the introduction of quantization. Debye's theory and Nernst's heat theorem are touched on. In passing, it should be pointed out that what is universally known as Avogadro's number in the United States is called Loschmidt's number in this book following the custom of some English and European writers.

Chapters 14 and 15, on conduction of electricity in gases and metals, superconductivity, cathode rays, thermionics, photoelectricity, etc., are well done and easy to follow. A good account of Bohr's theory of the hydrogen atom and the periodic table appear in Chapter 16.

Chapter 18 is a highly condensed account of quantum mechanics. The general reader with but little prior knowledge of physics and mathematics would find this chapter difficult to follow. In contrast, Chapters 19 and 20 on nuclear physics and cosmic rays should be understood without difficulty by the attentive reader.

In the Epilogue, Dr. Wilson shows how time after time physicists have been reluctant to entertain new and revolutionary ideas, which receive general acceptance only after thorough and prolonged study. This is a point well worth making for, to many persons who take a look at physics from the outside, the physicist often appears to be all too ready to entertain bizarre theories.

Unlike some writers on science for the general public Dr. Wilson makes no attempt to dazzle the reader by scintillating expressions, a procedure which too often results in giving him a misleading and superficial picture. On the contrary, Dr. Wilson gives a thoughtful and well-balanced account of the growth of physics during the last hundred years. The serious reader who can give adequate time and attention to the book will find his efforts well repaid.

A. L. HUGHES
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The Elements of Thermodynamics. CHARLES FABRY. Pp. 229 + vii. Frederick Muller, Ltd., London, 1951. Translated from the sixth French edition by G. F. Herrenden Harker.

Although published in French in 1928, this work has just become available to the majority of English speaking scholars. The volume in which it is contained is small, and the temperature of the presentation is reasonable so that the basic elements of classical thermodynamics can be contained at reasonable pressure. The thermodynamic concepts are introduced with exceptionally lucid explanations, and the book can be used with ease by students familiar with the elements of heat and the rudiments of partial differentiation. The relation between the thermodynamic theory and the supporting experiments is maintained throughout, and there are sections on experimental methods and on the evaluation of experimental results. Some of the more obvious extensions of the theories are sketched and left for detailed exploration by the reader, and the problems at the end of each chapter are chosen to produce real thought.

There is no material in the book on any theory of matter

beyond that which strictly belongs to classical thermodynamics, and there is very little material on the chemical aspects and the low temperature aspects of the field. If the book is chosen as a text for any course more demanding than an introductory course for physics students, it will have to be supplemented heavily with material from the special field. It would also be desirable to supplement the work with some elementary kinetic theory when it is used for science students. The book is of good mechanical construction and is easy to read and use. The translation is so well done that the original French scarcely shows.

RICHARD C. RAYMOND
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Creep of Metals. L. A. ROTHERHAM. Pp. 80. Figs. 24. The Institute of Physics, London, 1951. Price 15s.

Creep might be defined as the time rate of distortion of a metal at elevated temperatures. Any metallic structure (turbine blade, bearing, gear tooth, etc.), which is subjected to repeated stresses over a period of time and which is also maintained at a high temperature will creep; in other words, a structure which might not change its shape for one or even several applications of a load may, after thousands of such applications (no one of which exceeds the safe load for the structure), be deformed to such a degree that it will fail. So the designer of gas and steam turbines, steam boilers, reciprocating engines and the like has his problem further complicated.

Mr. Rotherham says that creep is partly "viscous" and partly crystalline slip. He says that the crystalline component becomes effective above a critical stress which is approximately the limit of proportionality in a short-time tensile test. While solute atoms complicate the picture and affect grain size, hardness, and work-hardening, their effect on recovery rate is the only one which influences creep.

There are two types of creep, transient and steady-state. The former is similar to crystalline plasticity involving a large number of atoms in the glide process, whereas the latter is amorphous plasticity involving the rearrangement of relatively few atoms. These two types occur in a manner analogous to that of transient and steady-state conditions in electrical and vibratory systems. It is somewhat more difficult to separate the transient from the steady-state creep since both are not only very complex within themselves but are also interrelated in a way which makes clear analysis extremely difficult.

Much of the theory behind creep is based on experimental data so that equations are largely empirical. This is understandable when one realizes how little is known about actual crystalline behavior in metals. However, the problem of creep is one which the physicist might consider more seriously in the future. Generally, physicists do not like to become involved with what they consider engineering problems. Yet here is one area where only a good knowledge of physics can hope to offer any serious assistance. Mr. Rotherham recognizes this and tries to maintain the physics approach. For the reader who has already served some apprenticeship in elasticity and plasticity, this little volume will prove very valuable.

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An Introduction to Electron Optics. L. JACOB. Pp. 150. John Wiley and Sons, Inc., New York, 1951. Price $2.00.

The stated purpose of the Methuen Monographs on physical sciences is "to supply readers of average scientific attainment with a compact statement of the modern position in each subject." Dr. Jacobs adds that his book is "an attempt to state the principal laws involved when considering the motion of charged particles through electric and magnetic fields." Within this framework, the author has, in my opinion, provided a worthy addition to the literature in electron optics.

Electron Optics is compact (137 small pages of text), well illustrated (47 figures, mostly line drawings), and modern (about half the 200 references, grouped by subject matter, are to work published in the past ten years). The laws of motion of charged particles are stated in terms of Fermat's principle and the index of refraction of charged particles in electric and magnetic fields. Analogies with light optics are not labored but are used when there results a simplification of the discussion of equivalent electron lenses. Axial fields occupy the majority of available space though some reference is made to special cases of motion in transverse fields. A good presentation is made of sources and corrections of aberrations in axial field electron lenses.

In summary the material presented might simply be described as suitable for *Reviews of Modern Physics* in the portions of electron optics treated. Like some articles in that journal, it has the difficulty for readers whose "average attainment" may not be in the field discussed that it reviews and introduces without evaluating. Thus often those of us to whom the material is directed are left without crutch or axe in our pursuit of more complete information in some phase of the subject reviewed. This book is sufficiently complete in axial field optics to leave this reader satisfied.

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Elements of Modern Physics. W. C. MICHELS AND A. L. PATTERSON. Pp. 659+x. D. Van Nostrand Company, New York, 1951. Price $6.50.

A casual reading of the table of contents and a flip through the pages could easily deceive one into thinking that this textbook is not the well-thought-out, reasonably comprehensive, and somewhat sophisticated book that it actually is. This is a general physics text written from a modern point of view and is not meant for the usual course in "Modern Physics." It should be suitable for all classes of students except engineers and others in the more technical programs.

The unorthodox arrangement and grouping of topics which are used are the result of the authors' aim to present physics in a straightforward manner using the viewpoint

of the professional physicist since they feel quite strongly that this "offers the best introduction to any student, whether or not that student intends to follow that field as a career." To accomplish this aim, they give coherent treatments of similar subjects, pointing out their differences as well as their similarities—for example, fields of all types (gravitational, electric, magnetic) are discussed in a single chapter, as is wave motion in another. Naturally, this does violence to the usual ordering based on either a conventional or a historical approach. An illustration is the introduction of the photon in Chapter 3 where it is used to clarify the meaning of a velocity measurement by interpreting the usual methods of measuring the velocity of light as a time-of-flight method for groups of particles. Similarly, the discussion of rotational motion and the complete treatment of statics is deferred until about the middle of the book. The ordering of the last half-dozen chapters does, however, seem somewhat jumbled, and it would be interesting to know the reasons for that particular choice.

The textbook is well written in a lucid and pleasant style. Difficult points and topics are not glossed over, and when approximations are used, the authors are careful to point them out and explain their effect on the limits of validity of further discussions and applications. Any impression that this is a grim book can be quickly dispelled, however, by noting the novel application of the Schmoo on p. 295.

An excellent feature is the almost exclusive use of MKS units—the simplification and freedom from wasted effort and confusion (for both instructor and student) which this provides in electromagnetism is well known. The theories of both special and general relativity are discussed in a refreshingly calm way which is in contrast to the more common treatments which concentrate on their paradoxical and "mystical" aspects.

Almost all of the illustrations are drawings which are notable for their clarity and aptness. The numerous examples are placed at the end of the appropriate section rather than being collected at the end of each chapter. Rougly a third of them are derivations which extend the material covered in the text; answers to the odd-numbered examples are provided. The printing and binding are uninspired but satisfactory.

Several premedical students who have been asked their opinion of this book as a possible text have been very favorably impressed by it. It is evident that a lot of hard work and careful thought has been devoted to this excellent book and it deserves serious consideration.

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Fundamentals of Physics. HENRY SEMAT. Pp. 849+xxv. Rinehart and Company, New York, 1951. Revised Edition. Price $6.00.

The 1951 edition of this textbook is much more than a revision of the 1945 edition, which was designed for an abbreviated course in elementary physics. In the new book several additional chapters have been inserted and added content is given to many others, the greatest expansion being in mechanics, heat, wave motion, and sound. Use is made of the MKS system of units as well as the cgs and the British systems to which the earlier edition was confined. There has been considerable rearrangement in the interest of clarity and teachability. A new section, Part Six, Atomic Physics, is formed by grouping together three previously scattered chapters. The 13-page glossary is retained in the new edition.

Professor Semat has had not only the benefit of his own many years of successful teaching, but he has also profited by the suggestions of many other experienced teachers. As a result the text reads smoothly, and each topic in general leads logically to the next. Each of the 35 chapters is followed by a list of carefully chosen questions, and a separate list of problems graded in the order of difficulty. Answers are given in the back of the volume for odd-numbered problems. Though the answers are not free from error, the proportion of errors is not high. The errors in textual content are few. Most of the 583 figures including the 200 new ones are excellent, as is indeed the entire format.

That the most painstaking writers occasionally let down their guards is illustrated by the treatment of the siphon, pp. 195–196. The pressure at the point A inside the short arm but at the *same* level as the outside free surface of the liquid is given as P_0-hdg, that is, the atmospheric pressure *diminished* by that resulting from the column of liquid on that side! Not unnaturally, a similar expression is given for the pressure at the point B at the same level as the outside free surface in the other arm. One might attribute the error to the misplacing of points A and B at the bottom instead of at the top of the columns together with the omission of a diaphragm or cutoff at the top between the two arms to create a static condition. However, the accompanying explanation takes a very different and equally fallacious line. Here the pressure in either arm is referred to as not only that resulting from the atmospheric pressure but "in addition" that resulting from the column of liquid in the arm!

It is regrettable that a 1951 textbook should not join in the overdue recognition of Celsius, discontinuing use of "centigrade" as voted by the Ninth General Conference on Weights and Measures. Meeting at Paris and Sèvres, October 12–21, 1948. U. S. representatives, E. U. Condon, Director, National Bureau of Standards; Eugene C. Crittenden, Associate Director, National Bureau of Standards.

Greek letter symbols are not introduced until the final chapter in mechanics, "so that the student will not be burdened with the need for learning new symbols as well as new concepts at the beginning of the subject." But it is a vastly more important consideration for the student to be able to associate each symbol used with a definite concept. Not only are Greek symbols replaced by English, but many standard English symbols are also switched. The result is just what might be expected. A is used to represent an angle, a point, and the force acting on the point, with horizontal and vertical components A_H and A_V, pp. 33–34. In almost adjacent sections the symbol u represents the constant velocity of a boat with respect to moving water (or the constant velocity of an airplane with respect to

moving air), and the initial velocity of a body moving with accelerated motion, including the case of the projectile with horizontal and vertical components u_1 and u_2.

The pre-emption of d as symbol for distance—it is also used for density—is especially unfortunate. Although the calculus would under no conditions be corequisite for a course using this text, there will still be students in the class taking the calculus concurrently; and if the instructor is fully alive there will be noncalculus students eager to know how elementary calculus methods can simplify the derivation of many of the formulas of mechanics. With d representing distance in the student's mind, he must take an unnecessarily large mental hurdle to grasp the significance of d standing as a prefix to indicate an infinitesimal increment of distance or of any other kind of variable.

In Chapter 10, rotational motion, Greek letter symbols make their first appearance. τ now stands for torque, for which no symbol has previously been used, though one is needed in studying equilibrium of rigid bodies. But in magnetism U is used for torque and τ for period of vibration. H_F and H_V are the symbols for heat of fusion and heat of vaporization, while H is total heat absorbed or rejected. Thus in any formula involving these quantities, H_F and H_V must have mass coefficients while H does not, introducing a lack of symmetry of symbolism where symmetry would be suggested.

In almost adjoining sections n represents not only the number of moles of a gas present and the number of molecules per unit volume (in accord with standard practice), but also the ratio of the specific heats of a gas. In magnetism, the Greek symbol ϕ for flux is shunned, for which N must serve, thus necessitating n for total number of loops. The concept of number of revolutions per second is not introduced.

Banning of Greek letter symbols is doubtless responsible for the failure to introduce the notation ΣF_x, ΣF_y or ΣX, ΣY, thereby losing an important standardizing device in problems in statics and dynamics. Instead, in dynamics the symbol F is used for the resultant, e.g., p. 80, for which the substitution ma is later made. Introducing F in this way is undesirable as it has every appearance of being another force.

The textbook makes no use of the idea of an increment. To be sure, the idea is introduced in the presentation of velocity varying both in magnitude and direction, pp. 44, 45, but even here the notation is not such as a student of calculus could approve. The concept, after being thus introduced, is completely dropped. The procedure followed is to represent any change, however small, as the final value minus the initial value. The method is unrealistic in that in most cases the experimentalist measures directly the increment itself; it is cumbersome to write or to say; it is more of a handicap than an aid to the student's thinking processes. Thus, the thermal expansion of a metal rod is always of the form $L-L_i$, pp. 258–261; in obtaining the acceleration in uniform circular motion, p. 101, we do not have ΔV, the incremental velocity which must be added to the initial value to obtain the final value, but V_B-V_A following the rule for subtracting vectors by reversing the direction of V_A and adding—a far less direct process. The method is used in an area integration (p. 282) to establish the equivalence of the area between volume limits under a Boyle's law curve on a PV diagram with the work done by the gas in expanding isothermally between these limits.

It would seem that the sacrifice of elegant and concise techniques was occasioned by an effort to avoid any resemblance to the calculus, since students poorly prepared in mathematics can easily become terrified if they believe that calculus methods are being used in the course. But is it not better, even in a course for the general student, to direct the primary appeal to those who are mathematically minded? Such students are not confined to those who have had advanced courses in mathematics; they even include many who have tried to avoid contact with mathematics after exposure to the sort of mathematical instruction too often given, and who enter the physics class with grave misgivings because of its mathematical content. The designation, orderly minded students, might be a more descriptive characterization. As for students who are unwilling or unable to do a small amount of analytical thinking, they are certain to become liabilities in a physics department, however much we cramp our procedures in an effort to coddle them.

The reviewer is well aware that a really systematic presentation of physics, especially mechanics, for the general student, would not decrease the need for superior class instruction,—a commodity in critically short supply. But is this not the essential route toward increasing the supply of able teachers? All of us must realize by now that we cannot depend on professional courses in physics either in the School of Engineering or the School of Education for the training of superior teachers. Unless we devote more effort to the development of future teachers, students who do not go into applied physics courses will soon be drifting into general education courses in physical science. What sort of preparation would our future high school teachers of physics then be obtaining?

The amount of space that has been devoted to adverse criticism is perhaps rather considerable. Even so, in the reviewer's estimation the 1951 edition of Semat, *Fundamentals of Physics*, has few peers among currently available texts for the general student. We are adopting it.

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